# THE MOTION CHARACTERISTICS OF A CYLINDER VEHICLE IN THE OBLIQUE WATER-EXIT PROCESS 

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## 1 Introduction

With the development of computer numerical technology, there are more and more numerical simulation methods (NSM) about water-exit movements of the vehicle [1]-[9].
In the article [1], the research is conducted on the evolvement process of the complex multiphase flow, flow field structure and characteristics of fluid dynamics in the motor nozzle and launching tube of the vertical launching of a direct igniting underwater missile by using NSM. The water-exit


#### Abstract

: The hydrodynamic model of a vehicle exiting the water surface obliquely has been analyzed. The analyzed object is a cylinder vehicle and its motion characteristics. Two methods have been used to simulate the water-exit process under the same conditions: Numerical Simulation Method (NSM) and Theoretical Model Solution Method (TMSM). The comparison results of the two methods can validate the hydrodynamic model founded in this paper. Different initial angles and different initial velocities have been simulated by this hydrodynamic model and the numerical simulation has been analyzed. The analysis reveals the rule of change of altitude and position of the vehicle in the water-exit process, and its motion after it exits the water surface. This paper explains why it is more difficult for the vehicle to exit the water obliquely than vertically. The results show that the hydrodynamic model of the water exiting vehicle can be used to research the exiting water motion characteristics. The models simulate the physics of motion realistically and this hydrodynamic model can be used as a foundation for the future research of the stability and control of a vehicle exiting the water.


movement of the semi-infinite cylinder has been calculated by using NSM with VOF in reference [6], and the relationship between the Weber and Froude numbers and the shape of the free surface has been studied. The cavitation multiphase flow and hydrodynamic force evolution rule during the rising process of the vertical launching submarinelaunched missile has been studied by using NSM and dynamic mesh technique in reference to [7]. The rule of change of the added mass while the spherical object is coming out of water is analyzed in reference to [8].

[^0]The benefits of NSM are:
(1) Not only can it obtain the motion track of the vehicle during a certain period, but also calculate the force and the state of motion of the object. For example, in reference to [1], not only can the motion parameter of the missile be calculated but also the distribution of liquid flow-field, the evolution of multiphase flow and the flow field structure in the launching process.
(2) The pressure at any time and any location point can be obtained clearly. For example, the pressure distribution of different head shapes can be obtained while the effect of different head shapes and pitching angles on the water-exit flow field is studied by the numerical method in reference to [9].
(3) A variety of complex phenomena can be studied. For example, in reference to [10], the effects of wave on exceeding water process of underwater vehicle have been studied.

But the disadvantage of NSM is also very obvious:
(1) A large numerical mesh with hundreds of millions of cells needs great computer power.
(2) The uncertainty of the calculated results.

The calculated results are closely related to the way of mesh rezoning and the choice of turbulence model. The result may diverge or go wrong by inappropriate mesh rezoning and turbulence model.
As a result of the limitation of the above 2 points, NSM is usually used to simulate a certain working condition rather than various working conditions in a large area indispensible for a search algorithm.
Besides that, it is not necessary to pay attention to the fluid distribution and the force detail on the local surface of the vehicle while it exits the water surface while studying the motion characteristics. The theoretical model solution method (TMSM) is adequate. This method is able to use a straightforward theoretical model to study the motion characteristics of the water exiting vehicle by ignoring the details.
This article aims at studying the motion characteristics of the water-exit process and TMSM is used to study this problem. At first, this paper mainly focuses on a cylinder vehicle. The
hydrodynamic model of water-exit is built by introducing the variable buoyancy, added mass and wetted surface area. The pitching moment caused by an incidence angle in the water-exit process has been considered in this model. Then, this model is verified with NSM. After that, the simulation of the water-exit process under different conditions is performed according to the factual situation. At last, the motion characteristics of the cylinder vehicle when exiting the water obliquely are analyzed.

## 2 The model

### 2.1 Shape of the vehicle

Peaked arch shape whose vertex angle is $30^{\circ}$ is designed as the head of the cylinder vehicle ${ }^{[11]}$.The tail is designed as a linear cutting tail. The contour of the vehicle is shown in
Figure 11.


Figure 1. Contour of the vehicle (unit: m)
The length of the vehicle is $L=5.33 m$, and the density is $\rho_{0}=1.0 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}$.

In Figure $1 r_{t}=\frac{0.2665}{2 \cdot \sin 15^{\circ} \cdot \sin 15^{\circ}}$.

$$
R(l)= \begin{cases}0 & \mathrm{U}_{1} \\ 0.2221 l+0.13325 & \mathrm{U}_{2} \\ 0.2665 & \mathrm{U}_{3} \\ \frac{0.533}{2}-r_{t}+\sqrt{r_{t}^{2}-\left(l+\frac{0.2665}{\tan 15^{\circ}}-5.33\right)^{2}} & \mathrm{U}_{4}\end{cases}
$$

The range $\left[\begin{array}{l}\mathrm{U}_{1} \\ \mathrm{U}_{2} \\ \mathrm{U}_{3} \\ \mathrm{U}_{4}\end{array}\right]=\left[\begin{array}{l}l \leq 0 \text { or } l>5.33 \\ 0<l \leq 0.6 \\ 0.6<l \leq 5.33-\frac{0.2665}{\tan 15^{\circ}} \\ 5.33-\frac{0.2665}{\tan 15^{\circ}}<l \leq 5.33\end{array}\right]$.

### 2.2 Force analysis

The body- axis coordinate system is built with the origin at the center of gravity, as shown in Figure 22.


Figure 2. Force analysis of the vehicle
The gravity $\mathbf{G}$, the buoyancy $\mathbf{B}$ and the fluid force $\mathbf{F}$ are forces acting on the vehicle.
The fluid force $\mathbf{F}$ consists of three components, the ideal fluid force, the viscous fluid force and the force caused by the incidence angle.
According to the ideal fluid force theory, the ideal fluid force of the water-exit process can be calculated:

$$
\begin{aligned}
{\left[\begin{array}{c}
R_{i x} \\
R_{i y} \\
M_{i z}
\end{array}\right] } & =-\lambda\left[\begin{array}{l}
\mathrm{d} v_{x} / \mathrm{d} t \\
\mathrm{~d} v_{y} / \mathrm{d} t \\
\mathrm{~d} \omega_{z} / \mathrm{d} t
\end{array}\right]-\left[\begin{array}{ccc}
0 & -\omega_{z} & 0 \\
\omega_{z} & 0 & 0 \\
0 & 0 & 0
\end{array}\right]\left\{\lambda\left[\begin{array}{l}
v_{x} \\
v_{y} \\
\omega_{z}
\end{array}\right]\right\} \\
& -\left[\begin{array}{lll}
0 & 0 & 0 \\
0 & 0 & 0 \\
-v_{y} & v_{x} & 0
\end{array}\right]\left\{\lambda\left[\begin{array}{l}
v_{x} \\
v_{y} \\
\omega_{z}
\end{array}\right]\right\}
\end{aligned}
$$

In the formula (2), $R_{i x}$ and $R_{i y}$ represent the force for the ideal fluid exerted on the X axis and Y axis of the vehicle, respectively; $M_{i z}$ represents the moment for the ideal fluid exerted on the vehicle's Z axis; $v_{x}$ and $v_{y}$ represent the velocity components of the vehicle acting in X axis and Y axis; $\omega_{z}$ represents the rotational speed of the vehicle's $Z$ axis.
According to the vehicle's symmetry, it follows that $\lambda_{12}=\lambda_{16}=\lambda_{21}=\lambda_{61}=0$, and $\lambda_{26}=\lambda_{62}$. The formula (2) can be simplified as

$$
\left\{\begin{align*}
R_{i x} & =-\lambda_{11} \cdot \mathrm{~d} v_{x} / \mathrm{d} t+\omega_{z}\left(\lambda_{22} v_{y}+\lambda_{26} \omega_{z}\right)  \tag{3}\\
R_{i y} & =-\lambda_{22} \cdot \mathrm{~d} v_{y} / \mathrm{d} t-\lambda_{26} \cdot \mathrm{~d} \omega_{z} / \mathrm{d} t-\omega_{z} \lambda_{11} v_{x} \\
M_{i z} & =-\lambda_{62} \cdot \mathrm{~d} v_{y} / \mathrm{d} t-\lambda_{66} \cdot \mathrm{~d} \omega_{z} / \mathrm{d} t+v_{y} \lambda_{11} v_{x} \\
& -v_{x}\left(\lambda_{22} v_{y}+\lambda_{26} \omega_{z}\right)
\end{align*}\right.
$$

The force caused by viscous fluid and the angle of attack is:

$$
\left\{\begin{array}{l}
R_{\mu x}=-C_{\mu x 0} \cdot \frac{1}{2} \rho S v^{2}  \tag{4}\\
R_{\mu y}=C_{y \alpha} \alpha \cdot \frac{1}{2} \rho S v^{2} \\
M_{\mu z}=m_{z \alpha} \alpha \cdot \frac{1}{2} \rho S v^{2} L-R_{\mu y} \cdot \frac{1}{2}\left(L-x_{a}\right)
\end{array}\right.
$$

In this formula, $R_{\mu x}$ is the resistance caused by the viscosity of the fluid; $R_{\mu y}$ is the lift force caused by the angle of attack; $M_{\mu z}$ is the pitching moment caused by the angle of attack; $\rho$ is the density of the water; $v$ is the velocity of the vehicle; $\alpha$ is the angle of attack; $L$ is the length of the vehicle; $C_{\mu x 0}$ is the zero-list resistance factor; $C_{y \alpha}$ is the factor by which the angle of attack can create the lift; $m_{z \alpha}$ is the factor of the pitch moment caused by the attack angle, which can be ignored, and here it is set $m_{z \alpha}=0$.
$R_{\mu y} \cdot \frac{1}{2}\left(L-x_{a}\right)$ is the pitch moment caused by the lift which is brought about by the attack angle in the water-exit process.

### 2.3 Other relevant parameters

(1) added mass

The immersed volume and the added mass decreases gradually during the water-exit movement of the vehicle. Profile method has been used to calculate the added mass [12].

$$
\left\{\begin{array}{l}
\lambda_{33}=\pi \rho \int_{0}^{L-x_{a}} R^{2}(x) d x  \tag{5}\\
\lambda_{23}=\pi \rho \int_{0}^{L-x_{a}} R^{2}(x) x d x \\
\lambda_{66}=\pi \rho \int_{0}^{L-x_{a}} R^{2}(x) x^{2} d x
\end{array}\right.
$$

Since the added mass $\lambda_{11}$ is as small as several percentage points for the cylinder vehicle, it is set to zero: $\lambda_{11}=0$.
(2) Buoyancy and center of buoyancy

The buoyancy changes with the water-exit distance $x_{a}$. Here, the effect of the variability of the free surface is ignored. Therefore, the buoyancy can be calculated as:

$$
\begin{equation*}
B=\pi \rho g \int_{0}^{L-x_{a}} R^{2}(x) d x \tag{6}
\end{equation*}
$$

The location of the center of gravity (the buoyant center when the vehicle is fully immersed):

$$
\begin{equation*}
x_{0}=\frac{\int_{0}^{L} R^{2}(x) x d x}{\int_{0}^{L} R^{2}(x) d x} \tag{7}
\end{equation*}
$$

and the buoyant center is:

$$
\begin{equation*}
x_{b 0}=\frac{\pi \rho g \int_{0}^{L-x_{a}} R^{2}(x) x d x}{B} \tag{8}
\end{equation*}
$$

Then, the distance between the mass center and the buoyant center is:

$$
\begin{equation*}
x_{b}=x_{0}-x_{b 0} \tag{9}
\end{equation*}
$$

(3) Wetted surface area

The wetted surface area can be calculated by the integral method:

$$
\begin{equation*}
S=2 \pi \int_{0}^{L-x_{a}} R(x) d x \tag{10}
\end{equation*}
$$

(4) Moment of inertia

The moment of inertia of the vehicle can be calculated by the integral method according to the parallel axis theorem and the moment of inertia theorem of the disc shape.

$$
\begin{equation*}
J=\pi \rho_{0} \int_{0}^{L} R^{2}(x)\left[1 / 4 R^{2}(x)+\left(x-x_{0}\right)^{2}\right] d x \tag{11}
\end{equation*}
$$

The moment of inertia can be calculated by numerical integration: $J=1939.62 \mathrm{~kg} \cdot \mathrm{~m}^{2}$.
(5) Volume

The integral used to calculate the volume is:

$$
\begin{equation*}
V=\pi \int_{0}^{L} R^{2}(x) d x \tag{12}
\end{equation*}
$$

and the result is $V=1.04 \mathrm{~m}^{3}$.

### 2.4 The hydrodynamic model

The hydrodynamic model of the vehicle can be set, according to the above force analysis

$$
\left\{\begin{array}{l}
R_{i x}+R_{\mu x}+(B-G) \sin \theta+T=m\left(\frac{d v_{x}}{d t}-v_{y} \omega_{z}\right)  \tag{13}\\
R_{i y}+R_{\mu y}+(B-G) \cos \theta=m\left(\frac{d v_{y}}{d t}+v_{x} \omega_{z}\right) \\
M_{i z}+M_{\mu z}-B x_{b} \cos \theta=J \cdot \frac{d v_{y}}{d t}
\end{array}\right.
$$

Some other auxiliary formulae:

$$
\left\{\begin{array} { l } 
{ v = \sqrt { v _ { x } ^ { 2 } + v _ { y } ^ { 2 } } }  \tag{14}\\
{ \omega _ { z } = \frac { d \theta } { d t } } \\
{ v _ { x } = \frac { d x _ { a } } { d t } }
\end{array} \quad \left\{\begin{array}{l}
\alpha=\arctan \left(v_{y} / v_{x}\right) \\
m=\rho_{0} \cdot V \\
\mathbf{G}=m g
\end{array}\right.\right.
$$

These equations from formula (1) to formula (14) can describe the water-exit process. When the initial condition is given, the water-exit process can be simulated.
The initial condition includes initial velocity ( $v_{x 0}, v_{y 0}$ ), initial angle $\theta_{0}$, rotation angular velocity $\omega_{z 0}$, and the distance in the axial direction between the head top point of the vehicle to the water level $x_{a 0}$. When the head top point is under the water level, it is defined to be negative; otherwise if it is above the water level, it is said to be positive.

## 3 The model verification

The above hydrodynamic model can be verified by comparison with the result calculated by NSM with the same initial conditions.

Simulation conditions are: initial velocity $v_{x 0}=10 \mathrm{~m} / \mathrm{s}, v_{y 0}=0$, initial angle $\theta_{0}=45^{\circ}$, rotation angular velocity $\omega_{z 0}=0$.
When the depth of the vehicle in the water is 2.5 times bigger than its diameter, the effect caused by the water surface wave can be ignored ${ }^{[13]}$. We set the initial distance in the axial direction between the head top point of the vehicle to the water level to be 3 times the diameter.
The result is shown in Figure 3, according to NSM [14].


Figure 3. The result of the water-exit process simulated by NSM

Comparison with simulation results:
(1) Axial velocity and radial velocity



Figure 4. Comparison of the axial and radial velocity

Figure 55 shows the axial load and radial load change curve, which do not contain the gravity. It can be observed through the simulation comparison that: (1) the result of the two methods basically tallies, and both of them can describe the trajectory and attitude of the water-exit vehicle. This proved the validation of the model built in this paper. (2) However, there are still some deviations between the results. The reasons are as follows:

1. TMSM ignored the effect caused by the water surface wave and the attached water. The water surface wave and the attached water can be seen on the surface of the vehicle from Figure 3.
2. NSM may cause errors in calculations because of the choice of the turbulence model, grid partition and initial parameters.
Figure 5 shows that the amplitude of the NSM curve is large while exiting the water surface, especially at the late process stages. The amplitude is influenced by the unsteady flow and the errors accumulated in the dynamic mesh calculation process stage.
(2) Axial load and radial load



Figure 5. Comparison of the axial and radial load
(3) Axial and radial displacement


Figure 6. Comparison of the axial and radial displacement
(4) Inclination angle


Figure 7. Comparison of the inclination angle

## 4 Model simulation

It can be found that the inclination angle decreases continuously in the water-exit process. The ending of the water-exit process is defined according to the change of the inclination angle as follows.
Success in water-exit process means that the vehicle can finish the water-exit process before the vertical axis of the vehicle moves horizontally. That is to say, the tail of the vehicle leaves the water surface ( $x_{a}>L$ ) before the inclination angle is reduced to zero.

Failure in water-exit process means that the vertical axis is horizontal before the vehicle finishes the water-exit process. That is to say, before the tail of the vehicle leaves the water surface $\left(x_{a}>L\right)$, the inclination angle is reduced to zero.
The time points at which the process can be judged as success or failure in water-exit process are defined as the end time of the simulation. The movement after that time is out of scope of this paper.
(1) Simulation 1, the effect of the initial angle Initial simulation condition: initial position $x_{a 0}=-1.599 \mathrm{~m}$; initial velocity $v_{x 0}=5 \mathrm{~m} / \mathrm{s}$, $v_{y 0}=0$; initial rotation angular velocity $\omega_{z 0}=0$. and the initial angle is gradually increased form $10^{\circ}$ to $90^{\circ}$ by $10^{\circ}$.
Figure 8 to Figure 10 show the simulation results. The fine dashed line in Figure 8 to Figure 10 means that at this initial condition, the vehicle will fail in water-exit process, while the heavy line means that
at this initial condition, the vehicle will succeed in water-exit process.


Figure 8. Height of center of gravity


Figure 9. Height of the vehicle's tail


Figure 10. Change of the inclination angle
Simulation analysis:
(1) The greater the initial angle, the longer the simulation time. The reason is that when the initial angle is greater, the axial component force of the gravity will be greater as well. The axial component
force has reduced the axial velocity. The distance between the head top point of the vehicle to the water level is equal.
(2) As is shown in Figure 8 to, the initial angle can be affected whether the vehicle succeeded or failed in the water-exit process. The appropriate initial angle for the vehicle in water-exit process is not the least angle or the greatest angle but the angle in between two vectors. The reason is that, when the initial angular deflection is too little, the vertical axis of the vehicle will move horizontally (the inclination angle is reduced to zero) under the action of buoyancy, before the tail of the vehicle leaves the water (Success in water-exit process); when the initial angle is too large, the vehicle must overcome more gravitational potential energy to exit the water surface. When the kinetic energy is not enough to overcome the gravitational potential energy, the vehicle will fail in water-exit process. For example, it can be seen that in
Figure 88, the center of gravity will be higher if the inclination angle is larger, but the tail of the vehicle is not out of the water surface at the corresponding time in Figure 9.
(3) The mass center and the vehicle's tail is increased gradually in height with the water-exit movement. The height will begin to decrease when the mass center and tail are in peak point. From the Figure 10Figure 10, it can be seen that the inclination angle is reduced to zero when the initial angle is $10^{\circ}, 20^{\circ}, 60^{\circ}$ and $70^{\circ}$. That means that the vehicle fails in water-exit process under these conditions. When the initial angle is between $30^{\circ} \sim 50^{\circ}$, the vehicle succeeds in water-exit process.
(4) The rule of change of the inclination angle. The inclination angle decreases gradually during the water-exit movement of the vehicle except for the vertical water-exit condition. The vehicle will succeed in water-exit process, when the angle is decreased to a certain value, if the initial angle is between $30^{\circ} \sim 50^{\circ}$. In other cases, the vehicle will fail in water-exit process.
(2) Simulation 2, the effect of the initial axial velocity
Initial simulation condition: position $x_{a 0}=-1.599 \mathrm{~m}$; initial angle $\theta_{0}=45^{\circ}$; initial rotation angular velocity $\omega_{z 0}=0$. And the initial axial velocity is gradually increased form $2 \mathrm{~m} / \mathrm{s}$ to $8 \mathrm{~m} / \mathrm{s}$ by $1 \mathrm{~m} / \mathrm{s}$.
Simulation analysis: the greater the initial axial velocity, the easier for the vehicle to finish the
water-exit process. Figure 13 shows that the greater the initial axial velocity, the smaller angular deflection.


Figure 11. Height of center of gravity


Figure 12. Height of the vehicle's tail


Figure 13. Change of the inclination angle

## 5 Analysis and Summary

The simulation shows that the water-exit process is affected by the initial condition. When the initial
inclination angle is too large or too little, or the initial axial velocity is too little, the vehicle will fail in the water-exit process. And the motion characteristics of the vehicle will be obtained in the simulation after the water-exit process. The vehicle has negative radial velocity and the inclination angle continues to decrease after the water-exit process, so the vehicle will drop into the water after the water-exit process unless some effective controlling measures are taken. This is the main reason why many vehicles choose the vertical water-exit way instead of the oblique way. But with the development of the control technology, the oblique way will be surely used in the future.
The hydrodynamic model of the water-exit process in this paper can simulate the water-exit process of the cylinder vehicle. The motion attitude of the vehicle can be received at any time of the process. The results of the two methods basically coincide. The movement rule and the condition in which the vehicle will succeed in water-exit process can be calculated from this study. TMSM can be calculated easily because it can encompass the main factors which direct the water-exit process and ignore some secondary factors. The model in this paper can provide the theory basis for the research of the stability analysis as well as steady-state and control characteristics estimation.

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