CONSTRAINED ROBUST MODEL PREDICTIVE CONTROL FOR TIME-DELAY DESCRIPTOR SYSTEMS WITH LINEAR FRACTIONAL UNCERTAINTY

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Abstract:

This paper addresses the robust model predictive control (MPC) for a class of time delay descriptor systems with linear fractional uncertainty and input constrains. The systems are transferred to the piecewise continuous descriptor systems and a piecewise constant control sequence is calculated by minimizing the worst-case quadratic objective function. At each sampling internal, by means of Lyapunov theory and optimization theory, the optimal problem with infinite horizon objective function is reduced to a convex optimization problem involving linear matrix inequalities. The sufficient conditions for the existence of the state feedback control are derived and expressed as linear matrix inequalities. Further, an iterative model predictive control algorithm is proposed for the on-line synthesis of state feedback controllers with the conditions guaranteeing that the closed-loop descriptor systems are regular, impulse-free and robust stable. Finally, a numerical example is presented to show the efficiency of the proposed approach.

1 Introduction

Model predictive control (MPC) [1-4] is a popular strategy in dealing with multivariable constrained control problems encountered in process industries which has attracted notable attentions in the control of dynamic systems and which plays an important role in control practices. MPC uses a system model to predict input future evolution along a given prediction horizon. The future predictions of the state, output, and input variables are used to minimize a given performance index, which is a cost function defining the optimization criteria used to determine the best possible control action sequence. In practice, real plants inherently include uncertainties that are to be considered in control design. The control design procedure has to guarantee stability, performance and robustness properties of closed-loop systems in the whole uncertainty domain, so it is extremely important for MPC to be robust when modeling uncertainty. Robust constrained MPC using linear matrix inequality (LMI) has been proposed by Ref [5], where the polytopic model and structured feedback uncertainty model were addressed. Their main idea is to use infinite horizon control laws to guarantee robust stability for state

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feedback control. Another paper by Ref. [6] presented the problem of designing a robust output/state model predictive control for linear polytopic systems with input constraints when all time demanding computations of output feedback gain matrices were realized off-line and when the actual value of the control variable was obtained through simple on-line computation of scalar parameters and respective convex combination of the computed matrix gains. Another work considered output feedback robust model predictive control for the quasi-linear parameter varying (quasi-LPV) system with bounded disturbance so that an iterative algorithm is proposed for the on-line synthesis of the control law via convex optimization [7]. References [8-10] addressed the robust model predictive control problems, giving sufficient conditions and expressions of robust model-based predictive control law, and analyzing the issues of feasibility and stability of the closed-loop systems with delay.

The descriptor system (also called a singular system) model is a natural representation of a dynamic system. It describes a larger class of systems than the normal system model does and has wide applications in process modeling. The research into descriptor systems has been a field of active researching [11-13]. In Ref.[11] a piecewise constant control sequence was calculated by minimizing the worst-case linear quadratic objective function for a class of uncertain descriptor systems. For uncertain descriptor systems with both state and input delays, the approximate solutions of optimal problems for infinite time interval and with quadratic performance index were calculated by means of Lyapunov theory and linear matrix inequalities (LMIs) technique, and the sufficient conditions for the existence of the robust model-based predictive control were given in Ref. [12]. Ref. [13] considered the stabilization of continuous time descriptor systems with respect to input constraints and presented a sampled-data model predictive control scheme. The stability of the closed-loop was achieved in a similar manner as for non-descriptor systems, utilizing a suitable terminal penalty term and a terminal region constraint.

The existing results are mainly concerned with the robust MPC of descriptor systems with norm-bound or ploytopic uncertainties. The research in this paper is focused on the extensions of the existing results to the more generalization uncertainty-linear fractional uncertainty, of which norm-bound uncertainty and positive-real uncertainty are its special cases that can be recast to linear fractional uncertainty. The main contribution of this paper is to present the robust model predictive control law for time delay descriptor systems with linear fractional uncertainty and input constraints, to analyze feasibility of the problem and provide all time demanding computations of state feedback gain matrices, guaranteeing the performance robustness and performance (guaranteed cost) over the whole uncertainty domain.

The paper is organized as follows. A problem formulation and preliminaries on a predictive state model as a descriptor system with linear fractional uncertainty is given in the next section. In section 3, the approach of robust state feedback predictive controller design using linear matrix inequality is presented. There is an example that illustrates the effectiveness of the proposed method which is discussed in section 4. Finally, some conclusions are given in the section 5.

Hereafter, the following notational conventions will be adopted: $\mathbf{R}$ denotes the set of real number; $\mathbf{R}^n$ denotes the n dimensional Euclidean space and $\mathbf{R}^{n\times n}$ is the set of all $n\times n$ real matrices. $\|x\|_2\ (x\in\mathbf{R}^n)$ and $\|x\|_Q\ (x\in\mathbf{R}^n, Q\in\mathbf{R}^{n\times n})$ denote $\|x\|=(x^{\mathrm{T}}x)^{1/2}$ and $\|x\|_Q = x^TQx$ respectively. Given a symmetric matrix P , the inequality $P>0 \quad (P\geq 0)$ denotes matrix positive definiteness (semi-definiteness) and I denotes the identity matrix of corresponding dimensions. The symbol * induces a symmetric structure in a matrix.

2 Problem statement and preliminaries

Consider a continuous-time descriptor system with delay and uncertainty

$$\begin{aligned} E\dot{x}(t) &= \bar{A}(t)x(t)+\bar{A}_1(t)x(t-h)+\bar{B}(t)u(t) \\ &= (A+\Delta A)x(t)+(A_1+\Delta A_1)x(t-h)+(B+\Delta B)u(t) \\ y(t) &= Cx(t) \\ x(t) &= \varphi(t), t\in[-h,0]. \end{aligned} \tag{1}$$

under the input constraints with Euclidean norm bounds: $\|u(t)\|_2 \leq u_{\max}, t\geq 0$,

where $x(t)\in\mathbf{R}^n$ is the state vector, $u(t)\in\mathbf{R}^m$ is the control input vector, $\varphi(t)$ is the continuous state initial function, $u_{\max}$ is a known real that denotes the allowable max value of the input vector $u(t)$'s Euclidean norm , the matrix $E \in \mathbf{R}^{n \times n}$ may be singular, and it is assumed that rank$(E) = r < n$, h is positive time-delay constants. The uncertainty set Ω :

$$\Omega = \{\begin{bmatrix}\bar{A}(t) & \bar{A}_1(t) & \bar{B}(t)\end{bmatrix} | \begin{bmatrix}\Delta A & \Delta A_1 & \Delta B\end{bmatrix} = D\bar{\Delta}(t)\begin{bmatrix}E_0 & E_1 & E_b\end{bmatrix},$$
$$\bar{\Delta}(t) = \Delta(t)[I - J\Delta(t)]^{-1}, \Delta^T(t)\Delta(t) < I, J^T J < I\}$$

A, A_1, $B, C, D, E_0, E_1, E_b, J$ are real constant matrices with appropriate dimensions, $\bar{\Delta}(t)$ is called linear fractional uncertainty and $\Delta(t) \in \mathbf{R}^{p \times q}$ is a time-varying matrix.

Remark 1: Input constraints are typically hard constraints since they represent limitation on process equipment (such as value saturation) and as such cannot be relaxed or softened.

MPC is an open-loop control design procedure. At each sampling time kT , plant measurements are obtained and a model is used to predict future inputs of a system. Using these predictions, m control moves $u((k+i)T, kT), i = 0,1,\cdots m-1$, are computed by minimizing a given cost function $J_p(k)$ over a prediction horizon p as follows:

$$\min_{u((k+i)T, kT), i=0,1,\cdots m-1} J_p(k)$$

where

$$J_p(k) = \sum_{i=0}^{p} \|x((k+i)T, kT)\|_{Q_1} + \|u((k+i)T, kT)\|_{Q_2}$$

p is output or prediction horizon, m is input or control horizon. The case $p = \infty$ is referred to as infinite horizon MPC.

Finite horizon control laws are known to have poor nominal stability properties [14]. Nominal stability of finite horizon control MPC requires imposition of a terminal state constraint ($x((k+i)T, kT) = 0, i = m$) and / or the use of the contraction mapping principle to tune Q_1, Q_2, m, p for stability. But the terminal state constraint is somewhat artificial since only the first control move is implemented. Thus, in the close loop, the states actually approach zero only asymptotically. Also, the computation of the contraction condition at all possible combinations of the constraints at the optimum of the on-line optimization can be extremely time consuming. On the other hand, infinite horizon laws have been shown to guarantee nominal stability [14], it is preferable to adopt the infinite horizon method that guarantees at least nominal stability.

We shall consider the case which is referred to as infinite horizon MPC for (1), i.e., control horizon and predictive horizon are all infinite. Let T be the fixed sampling interval. At sampling time kT for $k = 0,1,2\cdots$, plant measurements are obtained, then a predictive model is used to predict future behaviors of the system. Let $x(kT+\tau, kT)$ denote the predicted state at time $kT+\tau$, based on the measurements at sampling time kT , $x(kT, kT)$ refers to the state measured at sampling time kT , $u(kT+\tau, kT)$ is the control move for time $kT+\tau$ obtained by an optimization problem at time kT over the infinite prediction horizon. We assume that exact measurement of the states of the system (1) is available at each sampling time kT , i.e., $x(kT, kT) = x(kT)$.

The future behavior of the system is represented by future predictions of the state, output and input variables over the prediction horizon. For an infinite prediction, namely, the future predictions of the state, output and input variables are used to minimize a given performance

$$\min_{u(kT+\tau, kT), \tau \geq 0} J_\infty(k)$$

$$J_\infty(k) = \int_0^\infty (\|x(kT+\tau, kT)\|_{R_1} + \|u(kT+\tau, kT)\|_{R_2}) d\tau .$$

For the uncertain descriptor system with time-delay (1), at each sampling time kT , we discuss the minimization of a robust performance objective function as follows:

$$\begin{aligned} &\min_{u(kT+\tau, kT), \tau \geq 0} \max_{\begin{bmatrix}\bar{A}() & \bar{A}_1() & \bar{B}()\end{bmatrix} \in \Omega} J_\infty(k) \\ &s.t. \quad (1) \end{aligned} , \tag{2}$$

where

$$J_\infty(k) = \int_0^\infty (\|x(kT+\tau, kT)\|_{R_1} + \|u(kT+\tau, kT)\|_{R_2}) d\tau . \tag{3}$$

$R_1 > 0$, $R_2 > 0$ are symmetric weighting matrices.

This is a ‘min-max’ problem. The maximization is over the set Ω and corresponds to choosing that time-varying plants $\begin{bmatrix}\bar{A}(kT+\tau) & \bar{A}_1(kT+\tau) & \bar{B}(kT+\tau)\end{bmatrix} \in \Omega$, , $\tau \geq 0$, which, if used as a ‘model’ for predictions, would lead to the largest or ‘worst-case’ value of $J_\infty(k)$ among all the plants in set Ω. This ‘worst-case’ value is minimized over present or future control moves $u(kT+\tau, kT)$, $\tau \geq 0$.

We address the problems (2), and (3) first by deriving an upper bound on the robust performance objective. Then we minimize this upper bound with a constant state-feedback control law

$$u(kT+\tau,kT)=K(kT)x(kT+\tau,kT),\tau\geq 0. \qquad (4)$$

Next, we obtain a state feedback controller $K(kT)$, which makes the closed-loop system (1) be regular, impulse-free and asymptotically robust stable with input constraints $\|u(kT+\tau,kT)\|_2 \leq u_{\max},\tau\geq 0$.

In the receding horizon framework, only our very first computed control move $u(kT,kT)$ is implemented. At the next sampling time, the optimization (2), (3) is resolved with new measurements from the plant. Now we will review some lemmas for MPC.

Definition 1: Descriptor system $E\dot{x}(t)=Ax(t)+Bu(t)$ is stabilizable if there exists control law $u(t)=K(t)x(t)$ such that the closed-loop system is regular, impulse-free, and asymptotically stable.

Lemma 1[11]: Let orthogonal matrices $U=[U_1 \;\; U_2]$, $V=[V_1 \;\; V_2]$ be such that $E=U\begin{bmatrix}\Sigma_r & 0\\ 0 & 0\end{bmatrix}V^T$, from which it can be seen that $EV_2=0$, $U_2^T E=0$;the following items are true:

(1) Z satisfying $ZE^T=EZ^T\geq 0$ can be parameterized as $Z=EV_1W^TV_1^T+SV_2^T$ where $W\geq 0\in\mathbf{R}^{r\times r}$, $S\in\mathbf{R}^{n\times(n-r)}$.

(2) When $Z=EV_1W^TV_1^T+SV_2^T$ is nonsingular and $W>0$, then there exists $\hat{W}$ such that $(EV_1W^TV_1^T+SV_2^T)^{-T}=U_1WU_1^TE+U_2\hat{S}$ with $\hat{W}=\Sigma_r^{-1}W\Sigma_r^{-1}$ and $\hat{S}=U_2^T(EV_1W^TV_1^T+SV_2^T)^{-T}$.

Lemma 2[15]**:** A known descriptor delay system $E\dot{x}(t)=Ax(t)+A_1x(t-h)$ is regular, impulse-free and stable if there exist a matrix $Q>0$ and a nonsingular matrix P such that

$$\begin{aligned}&E^TP=P^TE\geq 0\\ &AP^T+P^TA+P^TA_1Q^{-1}A_1P+Q<0\end{aligned}.$$

Lemma 3[16]**:** (Schur complement) For given the symmetric matrices $S=\begin{bmatrix}S_{11} & S_{12}\\ S_{21} & S_{22}\end{bmatrix}$ and $S_{11}\in\mathbf{R}^{r\times r}$, the following three conditions are equivalent：

(1) $S<0$,

(2) $S_{11}<0$, and $S_{22}-S_{21}^TS_{11}^{-1}S_{12}<0$,

(3) $S_{22}<0$, and $S_{11}-S_{12}S_{22}^{-1}S_{21}^T<0$.

Lemma 4[17]: Assume the matrices $U=U^T$, $\Sigma=\Delta(t)(I-J\Delta(t))^{-1}$, $J^TJ<I$, $\Delta(t)^{\mathrm{T}}\Delta(t)<I$, U, J, H, E are known real matrices with appropriate dimensions, for all admissible $\Delta(t)$ *when* satisfying $\Delta(t)^T\Delta(t)\leq I$, an inequality

$$U+H\Sigma E+(H\Sigma E)^T<0$$

holds if and only if there exist some scalars $\varepsilon>0$ such that

$$\begin{bmatrix}\varepsilon U & H & \varepsilon E^T\\ * & -I & J^T\\ * & * & -I\end{bmatrix}<0 \qquad (5)$$

3 Main results

In this section, we discuss the infinite horizon MPC (IH-MPC) problem formulation for a class of descriptor systems. We begin with the robust IH-MPC problem without an input constraint, reduce it to a minimization of the worst-case objective function and then incorporate the input constraint. Finally, we show that the feasible receding horizon state-feedback control law robustly stabilizes the descriptor system over the uncertainty set Ω .

Consider a quadratic function:

$$V(x(t))=x(t)^TE^TPx(t)+\int_{t-h}^{t}x(s)^TQx(s)ds>0,$$

with $Q>0, E^TP=P^TE\geq 0$ and P is a nonsingular matrix.

At sampling time kT, suppose that $V(x(t))$ satisfies the following inequality for all $x(kT+\tau,kT)$, $u(kT+\tau,kT)$, and any uncertain plants $[\bar{A}(kT+\tau)\;\bar{A}_1(kT+\tau)\;\bar{B}(kT+\tau)]\in\Omega,$, $\tau\geq 0$:

$$\begin{aligned}&\frac{d}{d\tau}(V(x(kT+\tau,kT)))\leq\\ &-(\|x(kT+\tau,kT)\|_{R_1}+\|u(kT+\tau,kT)\|_{R_2})\end{aligned}, \qquad (6)$$

where $R_1>0, R_2>0$ are known weighting matrices.

For the robust performance objective function $J_\infty(k)$ to be finite, we must have $x(\infty,kT)=0$, and hence $V(\infty, kT) = 0$. Integrating both sides of the aforementioned inequality (6) from $\tau = 0$ to ∞, the following inequality (7) is obtained:

$$J_\infty(k) \le V(x(kT)) . \tag{7}$$

Thus, the robust MPC problem at time kT can be solved by minimizing the upper bound $V(x(kT))$, subjected to the imposed constraint (6):

$$\max_{[\bar{A}() \;\; \bar{A}_1() \;\; \bar{B}()]\in\Omega} J_\infty(k) \le V(x(kT)) \le \gamma , \tag{8}$$

where

$$V(x(kT)) = x(kT)^T E^T P x(kT) + \int_{-h}^{0} x(kT+\tau, kT)^T Q x(kT+\tau, kT) d\tau \tag{9}$$

This gives an upper bound on the robust performance objective. Thus, the goal of robust MPC algorithm has been redefined in order to synthesize, at each time step k, a constant state-feedback control law

$u(kT+\tau, kT) = K(kT)x(kT+\tau, kT)$, $\tau \ge 0$

to minimize this upper bound $V(x(kT))$:

$$\min_{u(kT+\tau,kT),\tau\ge 0} \max_{[\bar{A}() \;\; \bar{A}_1() \;\; \bar{B}()]\in\Omega} J_\infty(k) \le V(x(kT)) \le \gamma \tag{10}$$

where

$$V(x(kT)) = x(kT)^T E^T P x(kT) + \int_{-h}^{0} x(kT+\tau, kT)^T Q x(kT+\tau, kT) d\tau .$$

As it is standard in MPC, only the first computed input $u(kT, kT) = K(kT)x(kT)$ is implemented. At the next sampling time, the state $x((k+1)T)$ is measured, and the optimization problem is repeated so as to recompute K.

The following theorem gives LMI conditions for maintaining feasibility of the optimization problem (2) and for expressing the state feedback matrix $K(kT)$.

Theorem 1: For the known descriptor system(1) with linear fractional uncertainty $\bar{\Delta}(t)$, let $x(kT)$= $x(kT, kT)$ be the state of the descriptor system (1) measured at sampling time kT . At each sampling period $[kT, (k+1)T)$, the state feedback matrix $K(kT)$ in the controller (4) that minimizes the upper bound $V(x(kT))$ on the robust objective function is given by

$$K = Y^T (EV_1 W V_1^T + SV_2^T)^{-T} , \tag{11}$$

where $X_1 > 0, W > 0, Y, S$ and scalars $\gamma, \delta > 0$ are obtained from the solution of the following objective minimization problem:

$$\min_{\delta, W, X_1, M_1, S, Y} \gamma + tr(M_1) , \tag{12}$$

$$\begin{bmatrix} \gamma I & x^T(kT)V_1 \\ V_1^T x(kT) & W \end{bmatrix} \ge 0 , \tag{13}$$

$$\begin{bmatrix} M_1 & N_1^T \\ N_1 & X_1 \end{bmatrix} > 0 , \tag{14}$$

$$\begin{bmatrix} ZA^T + YB^T + AZ^T + BY^T & A_1 & \delta D & E_0^T + YE_b & Z & Z & Y \\ * & -X_1 & 0 & XE_1^T & 0 & 0 & 0 \\ * & * & -\delta I & -\delta J^T & 0 & 0 & 0 \\ * & * & * & -\delta I & 0 & 0 & 0 \\ * & * & * & * & -X_1 & 0 & 0 \\ * & * & * & * & * & -R_1^{-1} & 0 \\ * & * & * & * & * & * & -R_2^{-1} \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} u_{\max}^2 I & Y^T V_1 \\ * & W \end{bmatrix} \ge 0 , \tag{16}$$

where $Z = EV_1 W^T V_1^T + SV_2^T$, V_1, V_2 can be obtained by Lemma 1. N_1 can be calculated from $N_1^T N_1 = \int_{-h}^{0} x(kT+\tau, kT)^T x(kT+\tau, kT) d\tau$, $R_1 > 0, R_2 > 0$ are known weighting matrices.

Remark 2: Notice that K in (11) and the solutions $\gamma, \delta, W, X_1, M_1, S, Y$ to LMIs (12)–(16) depend only on the current state $x(kT)$ at sampling time kT . Strictly speaking, these variables should be denoted by $\gamma_k, \delta_k, W_k, X_{1k}, M_{1k}, S_k, Y_k$ to emphasize that they are computed at time kT . For notation convenience, we omit the subscript here.

Proof: At sampling period, $[kT, (k+1)T)$, define a Lyapunov-Krasovskii functional at $t \in [kT, (k+1)T)$ as (9):

$$V(x(kT)) = x(kT)^T E^T P x(kT) + \int_{-h}^{0} x(kT+\tau,kT)^T Q x(kT+\tau,kT) d\tau,$$

where $Q>0, E^T P = P^T E \geq 0$ and P is a nonsingular matrix.

If there exist scalars γ satisfying $x^T(kT)E^T Px(kT) \leq \gamma$, then

$$\max_{[\bar{A}\ \bar{A}_1\ \bar{B}]\in\Omega} x(kT)^T E^T Px(kT) \leq \gamma \tag{17}$$

Using Lemma 1 and Ref. [11], $x(kT)^T E^T Px(kT) \leq \gamma$ is equivalent to $x(kT)^T V_1 W^{-1} V_1^T x(kT) \leq 1$. Furthermore, $x(kT)^T V_1 W^{-1} V_1^T x(kT) \leq 1$ is equivalent to (13) by the Schur complement. Thus an invariant ellipsoid $\chi = \{z \mid z^T V_1 W^{-1} V_1^T z \leq 1\}$ is obtained for the predicted states of the uncertain system (1).

Remark 3: The maximization in (17) is over the uncertainty set Ω that can be used for predicting the future states of the system (1), this maximization leads to the 'worst-case' value of $x^T(kT)E^T Px(kT)$ at every instant of time $kT+\tau,\ \tau \geq 0$.

The second item in (9) may be reduced to

$$\int_{-h}^{0} x(kT+\tau,kT)^T Q x(kT+\tau,kT) d\tau$$
$$= \int_{-h}^{0} tr(x(kT+\tau,kT)^T X_1^{-1} x(kT+\tau,kT)) d\tau,$$
$$= tr(N_1^T N_1 X_1^{-1}) = tr(N_1^T X_1^{-1} N_1)$$

where $X_1^{-1} = Q$, assuming there exists a matrix M_1 such that $tr(N_1^T N_1 X_1^{-1}) < tr(M_1)$, then (14) holds by the Schur complement. We then minimize the 'worst-case' value of $V(x(kT))$ with a constant state-feedback control law (4) at every instant of time, $kT+\tau,\ \tau \geq 0$. So, $V(x(kT)) < \min\ \gamma + tr(M_1)$ is true and the problem (8) is implied to be $\min\ \gamma + tr(M_1)$.

From (1) and (4), the derivative of $V(x(kT+\tau,kT)$ along (7) can be derived as follows:

$$\begin{aligned}
&\dot{V}(x(kT+\tau,kT)) \\
&= \dot{x}^T(kT+\tau,kT)E^T Px(kT+\tau,kT) + x^T(kT+\tau,kT)P^T E\dot{x}(kT+\tau,kT) \\
&+ x^T(kT+\tau,kT)Qx(kT+\tau,kT) - x^T(kT+\tau-h,kT)Qx(kT+\tau-h,kT) \\
&= ((A+\Delta A)x(kT+\tau,kT) + (A_1+\Delta A_1)x^T(kT+\tau-h,kT) \\
&+ (B+\Delta B)Kx(kT+\tau,kT))^T Px(kT+\tau,kT) + \\
&x^T(kT+\tau,kT)P^T((A+\Delta A)x(kT+\tau,kT) + (A_1+\Delta A_1)x^T(kT+\tau-h,kT) \\
&+ (B+\Delta B)x(kT+\tau,kT)) + x^T(kT+\tau,kT)Qx(kT+\tau,kT) \\
&- x^T(kT+\tau-h,kT)Qx(kT+\tau-h,kT) \\
&\leq -x^T(kT+\tau,kT)(R_1 + K^T R_2 K)x(kT+\tau,kT)
\end{aligned} \tag{18}$$

(18) is also equivalent to

$$\begin{bmatrix} x(kT+\tau,kT) \\ x(kT+\tau-h,kT) \end{bmatrix}^T \begin{bmatrix} \Phi & P^T(A_1+\Delta A_1) \\ * & -Q \end{bmatrix} \begin{bmatrix} x(kT+\tau,kT) \\ x(kT+\tau-h,kT) \end{bmatrix} < 0.$$

Furthermore,

$$\begin{bmatrix} \Phi & P^T(A_1 + D\bar{\Delta}(t)E_1) \\ * & -Q \end{bmatrix} < 0$$

where

$$\Phi = \Phi_1 + (E_0 + E_b K)^T \bar{\Delta}^T(t) D^T P + P^T D\bar{\Delta}(t)(E_0 + E_b K)$$
$$\Phi_1 = (A+BK)^T P + P^T(A+BK) + Q + R_1 + K^T R_2 K.$$

Let $\Theta = \begin{bmatrix} \Phi_1 & P^T A_1 \\ * & -Q \end{bmatrix}$, (18) is implied to be

$$\Theta + \begin{bmatrix} P^T D \\ 0 \end{bmatrix} \bar{\Delta}(t) \begin{bmatrix} E_0 + E_b K & E_1 \end{bmatrix} + \begin{bmatrix} E_0 + E_b K & E_1 \end{bmatrix}^T \bar{\Delta}^T(t) \begin{bmatrix} P^T D \\ 0 \end{bmatrix}^T < 0. \tag{19}$$

By Lemma 4 and (18), there exist some scalars $\varepsilon > 0$ such that

$$\begin{bmatrix} \Phi_1 & P^T A_1 & P^T D & \varepsilon(E_0 + E_b K)^T \\ * & -Q & 0 & \varepsilon E_1^T \\ * & * & -\varepsilon I & \varepsilon J^T \\ * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{20}$$

The following inequality is derived by the Schur complement lemma:

$$\begin{bmatrix} \Phi_1 & P^T A_1 & P^T D & \varepsilon(E_0+E_b K)^T & I & I & K^T \\ * & -Q & 0 & \varepsilon E_1^T & 0 & 0 & 0 \\ * & * & -\varepsilon I & \varepsilon J^T & 0 & 0 & 0 \\ * & * & * & -\varepsilon I & 0 & 0 & 0 \\ * & * & * & * & -Q^{-1} & 0 & 0 \\ * & * & * & * & * & -R_1^{-1} & 0 \\ * & * & * & * & * & * & -R_2^{-1} \end{bmatrix} < 0 \tag{21}$$

Multiplying by $diag\{P^{-1}, Q^{-1}, \varepsilon^{-1}I, \varepsilon^{-1}I, I, I, I\}$ on the left of (21), multiplying by $diag\{P^{-1}, Q^{-1}, \varepsilon^{-1}I, \varepsilon^{-1}I, I, I, I\}$ on the right of (21), and defining $Z = P^{-T}, X_1 = Q^{-1}, \ \varepsilon^{-1} = \delta,$ $Y = ZK^T$, by Lemma 1, Z can be reconstructed by $Z = EV_1WV_1^T + SV_2^T$ so that (15) holds.

Physical limitation inherent in process equipment invariably imposes hard constraint on the manipulated variables $u(kT)$. We show how limits on the control signal can be incorporated into our robust MPC algorithm as sufficient LMI constraint. The basic idea of the discussion that follows can be found in Boyd et al. [16]. We present it here to clarify its application to our robust MPC setting and also to complete our exposition of the descriptor system (1).

At sampling time kT, consider the Euclidean norm constraint

$$\|u(kT+\tau, kT)\|_2 \le u_{\max}, \tau \ge 0 .$$

The constraint is imposed on the present and the entire horizon of future control variables, although only the first control move $u(kT,kT) = u(kT)$ is implemented. Following Ref.[16], we have

$$\begin{aligned} &\max_{\tau\ge 0} \|u(kT+\tau, kT)\|_2^2 \\ &= \max_{\tau\ge 0} \|Y^T Z^{-T} x(kT+\tau, kT)\|_2^2 \\ &\le \max_{z\in\chi} \|Y^T V_1 W^{-1} V_1^T z\|_2^2 \\ &= \lambda_{\max}(W^{-1/2} V_1^T Y Y^T V_1 W^{-1/2}) \end{aligned}$$

Furthermore, $W^{-1/2}V_1^T YY^T V_1 W^{-1/2} \le u_{\max}^2 I$, the input constraint holds. Pre-multiplying and post- multiplying $W^{1/2}$ on $W^{-1/2}V_1^T YY^T V_1 W^{-1/2} \le u_{\max}^2 I$, by the Schur complement lemma, we see that $\|u(kT+\tau,kT)\|_2^2 \le u_{\max}^2$, $\tau \ge 0$, if

$$\begin{bmatrix} u_{\max}^2 I & Y^T V_1 \\ * & W \end{bmatrix} \ge 0 .$$

So, the input constraint is translated to a sufficient LMI constraint.

The robust state feedback predictive controller $K = Y^T(EV_1WV_1^T + SV_2^T)^{-T}$ at sample period $[kT,(k+1)T)$, (15) is LMI with respect to $Y, Z \ \ X_1 > 0, \delta > 0$. The proof is completed.

In order to prove the closed loop system to be regular, impulse-free and asymptotically robust stable, we need to introduce the following lemma.

Lemma 5[5]: (Feasibility) Any feasible solution of the optimization (12)…(16) at time kT is also feasible for all times $\tau > k$, thus, if the optimization problem (12) is feasible at time k then it is feasible for all times $\tau > k$.

Theorem 2: If the optimization problems (12)…(16) and feasible solutions in the moment kT exist, thus: (a) there also exist feasible solutions in the NT moment $NT(N \ge k)$. (b) We get a piecewise state feedback control sequence $\{K_k\}_{k=0}^{\infty}$ when k changes from 0 to ∞. Therefore, the closed-loop system which is composed of piecewise state feedback control sequence $\{K_k\}_{k=0}^{\infty}$ is regular, impulse-free and asymptotically robust stable.

Proof: First, we show that the closed-loop system is regular and impulse-free, at sampling period $t \in [kT,(k+1)T)$, from (18), the following inequality holds

$$\frac{d}{d\tau}(V(x(t))) \le -(x(t)^T (R_1 + K^T R_2 K) x(t))$$

where $R_1 > 0, R_2 > 0$, so $\frac{d}{d\tau}(V(x(t))) < 0$ is true and $V(x(t)$ is a strictly decreasing Lyapunov-Krasovskii functional for the closed-loop system, which is bound blow by a position-definite function of $x(t)$, and the closed-loop system is asymptotically robust stable. (18) is implied to be

$$\begin{aligned} &\dot V(x(t) \\ &= \dot x^T(t) E^T P x(t) + x^T(t) P^T E \dot x(t) + x^T(t) Q x(t) \\ &\quad - x^T(t-h) Q x(t-h) \\ &= ((A+\Delta A)x(t) + (A_1+\Delta A_1)x^T(t-h) + (B+\Delta B)Kx(t))^T Px(t) \\ &\quad + x^T(t) P^T ((A+\Delta A)x(t) + (A_1+\Delta A_1)x^T(t-h) + (B+\Delta B)x(t)) \\ &\quad + x^T(t) Q x(t) - x^T(t-h) Q x(t-h) \\ &< 0 \end{aligned}$$

Then, the inequality

$$\begin{bmatrix} x^T(t+\tau) & x^T(t+\tau-h) \end{bmatrix} \begin{bmatrix} \Phi_2 & P^T(A_1+\Delta A_1) \\ * & -Q \end{bmatrix} \begin{bmatrix} x(t+\tau) \\ x(t+\tau-h) \end{bmatrix} < 0$$

is guaranteed or equivalently,

$$((A+\Delta A)+(B+\Delta B)K)^T P + P^T((A+\Delta A)$$
$$+(B+\Delta B)K)+Q+P^T(A_1+\Delta A_1)Q^{-1}(A_1+\Delta A_1)^T P<0 ,$$

where

$$\Phi_2 = ((A+\Delta A)+(B+\Delta B)K)^T P$$
$$+P^T((A+\Delta A)+(B+\Delta B)K)+Q .$$

By Lemma 2, the closed-loop system is regular, impulse-free and asymptotically robust stable.

In fact, the inequalities (13)…(16) are sufficient conditions for robust MPC synthesis problem of the system (1).

If $k=0$, the optimization problem (12)…(16) is feasible, i.e.,

$$\min_{\delta,W,X_1,M_1,S,Y} \quad \gamma + tr(M_1)$$

$$\text{s.t.} \begin{bmatrix} \gamma I & x^T(0)V_1 \\ V_1^T x(0) & W \end{bmatrix} \geq 0 ,$$

and (14)…(16) hold true. By Theorem 2, the optimization problem (12)…(16) is feasible at $k=1,2,3,\cdots$. The state feedback control sequence $\{K_k\}_{k=0}^{\infty}$ can be derived. If $k=0$, the optimization problem (12)…(16) is unfeasible for some descriptor systems with some initial state conditions, the obtained method in this paper cannot be used to synthesize these descriptor systems with some initial state conditions. So, it is not possible to extend the obtained results for all the state space.

The MPC scheme stated previously is summarized as follows. MPC Algorithm is described as follows:

Step 1. Set $k=0$

Step 2. Solve the convex programming problem (12) subject to (13)-(16) and compute by (11) to obtain a controller K.

Step3. Implement the control action $u(t)=Kx(t)$ for $t\in[kT,(k+1)T)$, computed control vector $u(t)$ is applied to the controlled plant, then measure the state $x(kT+T)$.

Step 4. Set $k=k+1$ and go back to step 2.

4 Experimental example

In this subsection, we present a numerical example that illustrates the implementation of the proposed robust MPC algorithm. This example also serves to highlight some of the theoretical results in the paper. For this example, LMI control toolbox software in the Matlab environment is used to compute the solution of the objective minimization problem.

Consider a descriptor system with the form of (1)

$$E=\begin{bmatrix}1&0\\0&0\end{bmatrix}, A=\begin{bmatrix}4&1\\2&3\end{bmatrix}, A_1=\begin{bmatrix}-0.3&0.1\\0.1&-0.3\end{bmatrix},$$

$$B=\begin{bmatrix}1&0\\0&2\end{bmatrix}, G=I=\begin{bmatrix}1&0\\0&1\end{bmatrix}, D=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix},$$

$$E_0=\begin{bmatrix}1&0\\0&1\end{bmatrix}, E_1=\begin{bmatrix}0.5&0\\0&0.5\end{bmatrix}, E_b=\begin{bmatrix}0&0\\0&1\end{bmatrix},$$

$$\Delta(t)=\begin{bmatrix}\sin t&0\\0&e^{-t}\end{bmatrix},\ \|u(kT+\tau,kT)\|_2<2$$

$$x(0)=\begin{bmatrix}0.5&-0.8\end{bmatrix}^T, R_1=I,\ R_2=I .$$

where $\Delta^T(t)\Delta(t)\leq I$, $h=0.5$, the sampling interval $T=0.3s$. By solving the optimization problem given in Theorem 1 via Matlab software, the states trajectories and control inputs trajectories of the descriptor system are shown in Fig.1 and Fig.2 when the time varying uncertainty is given by $\Delta(t)$.

From Fig. 1 and Fig. 2, we can observe that the proposed MPC algorithm for the descriptor system works well to asymptotically stabilize the descriptor system.

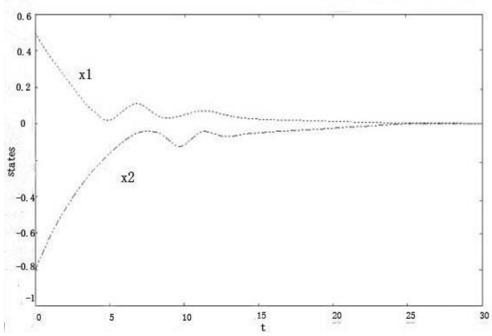

Fig. 1. Status of the closed-loop system

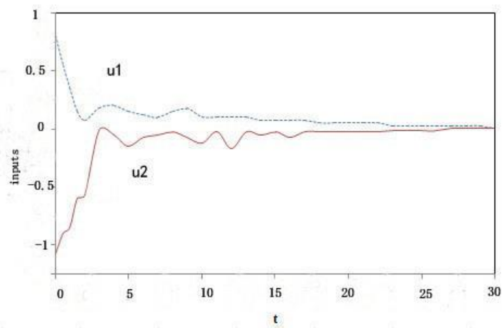

Fig. 2. Inputs of the closed-loop system

5 Conclusion

This paper has discussed the robust model-based predictive controller design methods for a class of uncertain descriptor systems with time delay subjected to an input constraint. The sufficient conditions in which robust model predictive controllers exist have been presented by Lyapunov stability theory, optimization theory and linear matrix inequality (LMI) method, a parameter notation of state feedback controllers have been obtained whenever these conditions have feasible solutions. Finally, a numerical example has been provided to demonstrate the applicability of the proposed approach. Linear fractional uncertainty is a more widely uncertainty than norm-bound uncertainty and positive-real uncertainty so that the research result in this paper has great significance for practical applications.

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