ANALYSIS OF THE SHEAR LAG EFFECT OF CANTILEVER BOX GIRDER

Y. Zhou*

School of Highway, Chang' an University, Xi' an 710064, Shanxi, China

ARTICLE INFO

***Article history*:**
Received: 21.11.2013.
Received in revised form: 06.03.2014.
Accepted: 12.03.2014.
Keywords:
Cantilever box girder
Shear lag effect
Minimum potential energy
Variation method
Shear lag coefficient

Abstract:

The theoretical solution method for the shear lag effect of cantilever box girder is solved through exerting the principle of minimum potential energy and combining the variation method, where the computation expressions of longitudinal displacement function and maximum angle displacement difference function are deduced and the computational formulas of the shear lag coefficient, additional bending moment and deflection are also deduced under condition that the cantilever box girder is acted on the concentrated force load and the uniformly distributed load. The simulated analysis model is built with the finite element method, and the analysis indicates that if the external load produces a constant shear flow within the section of the cantilever beam, the positive shear lag effect will be produced only; if the such load produces a varying or reverse shear flow within the section of the cantilever beam, the negative shear lag effect will be produced; the magnitude of the shear lag coefficient is proportional to the wide-span ratio; either the positive or negative shear lag effect will increase the structural deflection. The theoretical analysis is consistent with the analysis conclusion of the finite element method.

1 Introduction

Due to having strong bending resistance and torsional rigidity, good section integrity, and better construction adaptability etc., box girders are widely used in the bridge engineering, where big cantilever slabs and wide rid spacing box girders are further adopted with the increase of traffic volume [1-2]. However, due to the fact that a box girder is deformed by bending, the top plate and base plate of the box girder will be suffered from the impact of web shear deformation, and the internal stress of section will produce the phenomena of non-uniform distribution along the width of flange plate, which is called the shear lag effect [3-4]. As shown in Fig. 1 [5] (in which (a) shows the positive shear lag effect, and (b) shows the negative shear lag effect). The previous researches prove that the shear lag effect affect the strength and rigidity of structure [5-6]. Therefore, the research of the shear lag effect is necessary. There are many methods for researching the shear lag, and at present, the common methods

* Corresponding author. Tel.: +86 138 9283 5952, +86 29 82335910; fax: +86 29 82334773

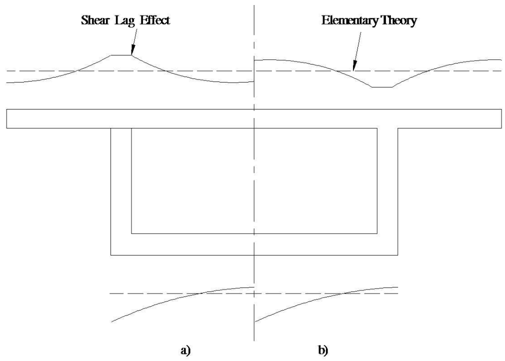

Figure 1. Shear lag effect of box section: (a) positive shear lag effect, (b) negative shear lag effect.

are as follows: (1) finite element method; It is mainly based on the three-dimensional shell and block theory, by which the fine three-dimensional finite element model of structure is created, to which the boundary condition and loading condition is exerted, of which the longitudinal stress distribution is obtained, upon which the shear lag effect is assessed [1, 3, 7, 8]; (2) analogy bar method: The structure of the box girder is simulated into the combination of the rod piece which is acted the axial force only and the sheet which is acted shear force only, and the equilibrium condition and deformation coordination relation are used for building the differential equation sets, so as to obtain the longitudinal stress of flange plate and analyze the shear lag effect [9]; and (3) energy variation method: By sing the principle of minimum potential energy and combining the variation method, the analytical solution of stress and deflection is obtained [10]. The energy variation method was first proposed by Reissner [11] and developed by many persons later, mainly including the method for the warping displacement function of shear lag in the correction energy method proposed by Wu and Zhang [12-13], which makes the solution more accurate; analysis on the shear lag effect of a cracked box girder by Cao [14-15]; research on the influence of the load effect position to the shear lag by Li [16], and Zhou [17], who proposed the analytical computational method of correcting the transverse displacement function on the basis of the distribution rule of the shear flow within the section of the box girder; Zhou [18-19] proposed a finite element method which matches with the current common beam element on the basis of the energy method, so that the solution is easier.

However, the researches above mainly concentrate on the simply supported and continuous beam, few researches on the shear lag effect of cantilever box girders which are widely used [3, 6]. At present, there are still many cantilever box girder bridges which are in service or being built, also, there are many bridges which are in cantilever state for a long period of time in the building process, i.e. large span continuous beams and rigid frame bridges, and there is significant difference in the stress state between these cantilever box girder bridges and the simply supported or continuous beams. Therefore, it is incomplete to not include the shear lag effect of cantilever box girders. What's more, more and more engineering practices indicate that the negative shear lag effect is not a special case, but a general one; in particular, for the cantilever box girder bridges, curved girder bridge or skew girder bridge, the negative shear lag effect is more obvious [20-22]. Like the positive shear lag effect, the negative shear lag effect will have influence on the safety of structure as well. However, the previous researches were concentrated more on the positive shear lag effect, few on the negative shear lag effect. This paper studies the shear lag of cantilever box girder bridges by exerting the principle of minimum potential energy and combining the variation method. This method can not only solve the positive shear lag effect, but also solve the negative shear lag effect. In the method, the warping displacement function is defined as fourth-degree parabola in order to make the solution more accurate, and influencing factors of shear lag effect are studies. The analytical method and other special techniques for calculating the shear lag of cantilever box girder are described. In addition, the finite element method is used for supporting analysis; and the results prove that the analysis results of the two methods are identical to each other.

2 Shear lag effect of cantilever beam obtained by the energy variation method

2.1 Basic assumption

As shown in Fig. 2, the origin of the coordinate system is the section centroid, and the distances of the upper and lower flange plates to y axis are z_u and z_b respectively; the thicknesses of the upper and lower flange plates are t_u and t_b respectively; and the web thickness is t_w, and the height of box girder is

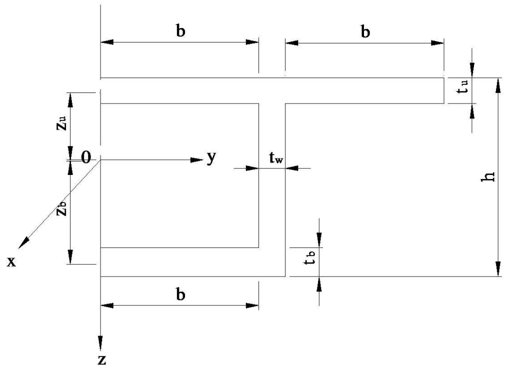

Figure 2. Box girder section and coordinate system.

h. Now, the following assumptions are made according to former researches:

1. Assume the longitudinal generalized displacement function of section:

$$u(x,y) = z_{u(b)}\left(w' + (1-\frac{y^4}{b^4})u(x) \right), \quad (1)$$

where $u(x)$ is the maximum angle displacement difference function, and w is the vertical deflection of the section centroid.

2. The web plate meets the plane section assumption of elementary beam theory under the effect of the symmetrical load. While computing the external potential energy, the longitudinal bending potential energy is considered only, and the transverse bending potential energy may be ignored.

3. The vertical pressing of the upper and lower flange plates and shear deformation out of the plate, belonging to the micro elements, may be ignored.

2.2 Deduction of basic differential equation

According to the principle of minimum potential energy, under the effect of the external force, there exists a group of data of the elastomer which is in the stable equilibrium state in all the displacements meeting the boundary condition. This group of displacement can make the total potential energy of whole system smaller, and at this time, the first order variation for the total potential energy of the system is zero. That is:

$$\delta\Pi = \delta(\bar{U} + \bar{V}) = 0\,, \quad (2)$$

where, Π is the total potential energy of system; $\bar{U}$ is the deformation potential energy of system; and $\bar{V}$ is the load potential energy of system.

The various deformation potential energies of box girder are solved as:

1. Web potential energy $\bar{U}_w$ of box girder:

$$\bar{U}_w = \frac{1}{2}\int_0^l EI_w \left(w''\right)^2 dx\,. \quad (3)$$

In the formula, E is the elasticity modulus of box girder materials; l is the computed span of box girder; and I_w is the moment of inertia of the web to the box section centroidal axis.

2. Strain energy of flange plate

From the assumption 1, it is obtained that the positive strain and shear strain of the upper flange plate of box girder may be computed according to the following formulas:

Bending strain

$$\varepsilon_u = \frac{\partial u\ (x,y)}{\partial x} = z_u\left(w'' + (1-\frac{y^4}{b^4})u'(x) \right). \quad (4)$$

Shear strain

$$\gamma_u = \frac{\partial u(x,y)}{\partial y} = \frac{-4z_u y^3}{b^4}u'(x)\,. \quad (5)$$

The strain energy of the upper flange plate is:

$$\bar{U}_u = \frac{1}{2}\int\int t_u\left(E\varepsilon_u + G\gamma_u{}^2\right)dxdy\,. \quad (6)$$

In the formula, G is the shear modulus of box girder materials. Equations (4) and (5) are applied in Eq. (6), it can be obtained:

$$\bar{U}_u = 2Et_u h_u{}^2 b\int(w''^2 + \frac{32}{45}u'^2 + \frac{8}{5}w''u' + \frac{16G}{7E}\frac{u'}{b^2})dx \quad (7)$$

Similarly, the strain energy of the lower flange plate is obtained:

$$\bar{U}_b = Et_b h_b{}^2 b\int(w''^2 + \frac{32}{14}u'^2 + \frac{8}{5}w''u' + \frac{16G}{7E}\frac{u'}{b^2})dx \quad (8)$$

Order $I_s=I_{su}+I_{sb}=4t_bh_u^2b+2t_bh_b^2b$ and $I=I_s+I_w$, so the strain energies for the upper and lower flange plates of the box girder are as follows:

$$\overline{U}=\overline{U}_u+\overline{U}_b$$
$$=\frac{1}{2}EI_s\int(w''^2+\frac{32}{14}u'^2+\frac{8}{5}w''u'+\frac{16G}{7E}\frac{u'}{b^2})dx \quad (9)$$

3. Potential energy of external load

When the box girder is bent, the external potential energy is:

$$\overline{V}=-\int M(x)\frac{d^2w}{dx^2}dx. \quad (10)$$

In the formula, $M(x)$ is the bending moment of section.

4. Energy equation solving

Equations (9) and (10) are applied in Eq. (2), it is obtained:

$$\delta\Pi=\int_0^l\left[M(x)+EIw''+EI_s\frac{4}{5}u'\right]dw''.$$
$$+\int_0^l\left[\frac{16}{7}GI_s\frac{u^2}{b^2}-\frac{32}{45}EI_su''-\frac{4}{5}EI_sw''\right]\delta udx.$$
$$+\left[(\frac{32}{45}EI_su'-\frac{4}{5}EI_sw'')\delta u\right]_0^l=0. \quad (11)$$

If Eq. (11) is true, the following equations are true:

$$M(x)+EIw''+\frac{4}{5}EI_su'=0, \quad (12)$$
$$\frac{16}{7}\frac{u}{b^2}GI_s-\frac{32}{45}EI_su'-\frac{4}{5}EI_sw'''=0, \quad (13)$$
$$EI_s(\frac{32}{45}u'-\frac{4}{5}w'')\delta u\Big|_0^l=0. \quad (14)$$

Order $n=\frac{1}{1-\frac{9I_s}{10I}}, k=\frac{1}{b}\sqrt{\frac{5Gn}{14E}}$, so Eq. (13) is simplified as:

$$u''-k^2u=-\frac{9nQ(x)}{8EI}. \quad (15)$$

The Eq. (15) is the second order nonhomogeneous differential equation, and the solution of it is:

$$u(x)=\frac{7n}{EI}\left(C_1\sinh(kx)+C_2\cosh(kx)+u^*\right). \quad (16)$$

In the formula, u^* is particular solution related to shear force $Q(x)$ of section, and coefficient C_1 and C_2 are determined by the boundary condition which is determined by the Eq. (14).

The analytical expression of the deflection obtained from Eq. (12) is as follows:

$$w''=-\frac{1}{EI}(M+M^F). \quad (17)$$

In the formula, M^F this is additional bending moment caused by the shear lag effect, and is to be solved by the following formula:

$$M^F=\frac{4}{5}EI_su'. \quad (18)$$

5. Shear lag coefficient $\lambda=\delta_x/\delta_0$ is defined as the coefficient of the shear lag effect, where, δ_x is the bending normal stress of section obtained according to the method of the paper, and δ_0 is the bending normal stress of section obtained according to the beam flexure theory. From Eq. (17) and Eq. (18), it is obtained that the shear lag coefficient at the junction of the flange plate and web is:

$$\lambda^e=1+\frac{4}{5}\frac{I_s}{I}\frac{EI}{M}u'. \quad (19)$$

The shear lag coefficient of the flange plate at the axis of symmetry is:

$$\lambda^c=1-(1-\frac{4}{5}\frac{I_s}{I})\frac{EI}{M}u'. \quad (20)$$

It is obtained that so long as the expression of $u(x)$ is produced and the shear lag effect at the various points for the various sections of box girder will be obtained. However, when $\lambda > 1$, the shear lag effect is the positive shear lag effect, and when $\lambda < 1$, the shear lag effect is the negative shear lag effect.

2.3 Shear lag effect solving of cantilever beam under the effect of specific load situation

1. Condition of the concentrated force under the span effect

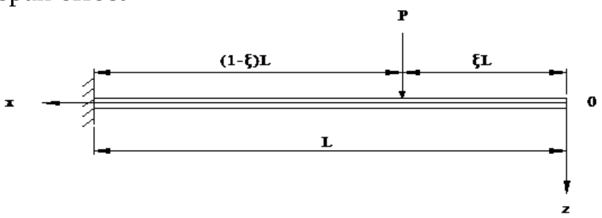

Figure 3. Cantilever bearing the concentrated load.

As shown in Fig. 3, the span of cantilever beam l, and a concentrated load P is acted at the position which is ξl away from the origin. It is computed and obtained: When $0 \le x \le \xi l$, $M(x) = 0$, $Q(x) = 0$ is applied in Eq. (16), the generalized displacement $u_1(x)$ is obtained:

$$u_1(x) = C_1 \sinh(kx) + C_2 \cosh(kx). \qquad (21)$$

When $\xi l \le x \le (1-\xi)l$, $M(x) = -P(x-\xi l)$ and $Q(x) = -P$ is applied in Eq. (16), the generalized displacement $u_2(x)$ is obtained as follows:

$$u_2(x) = \frac{7n}{EI}\left(C_3 \sinh(kx) + C_4 \cosh(kx) + \frac{9P}{56k^2}\right). \qquad (22)$$

The various coefficients obtained from the boundary condition are as follows:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{9P}{56k^2}\left(\cosh(k\xi l) + \frac{1}{\cosh(kl)} - \tanh(kl)\sinh(k\xi l)\right) \\ \frac{9P}{56k^2}\sinh(k\xi l) \\ -\frac{9P}{56k^2}\left(\frac{1}{\cosh(kl)} - \tanh(kl)\sinh(k\xi l)\right) \end{bmatrix} \qquad (23)$$

The shear lag coefficient obtained through further computing is as follows:

$$\lambda^e = 1 + \frac{4}{5}\frac{I_s}{I}\frac{EI}{M_y}\cdot\frac{9nP}{8EIk}\left[\sinh(k\xi l)\cosh(kx) - \frac{1}{\cosh(kl)} + \tanh(kl)\sinh(k\xi l)\cdot\sinh(kx)\right], \qquad (24)$$

$$\lambda^c = 1 - (1 - \frac{4}{5}\frac{I_s}{I})\frac{EI}{M_y}\cdot\frac{9nP}{8EIk}\left[\sinh(k\xi l)\cosh(kx) - \frac{1}{\cosh(kl)} + \tanh(kl)\sinh(k\xi l)\cdot\sinh(kx)\right]. \qquad (25)$$

Ordering

$$\sinh(k\xi l)\cosh(kx) - \frac{1}{\cosh(kl)} + \tanh(kl)\sinh(k\xi l)\cdot\sinh(kx) = 0. \qquad (26)$$

If x is obtained, the area for happening the negative shear lag will be obtained. Equation (26), which is a transcendental equation, may be applied for numerical solution.

2. Condition of uniformly distributed load under the effect of whole span

As shown in Fig. 4, the bearing magnitude of a certain cantilever beam is the uniformly distributed load of q.

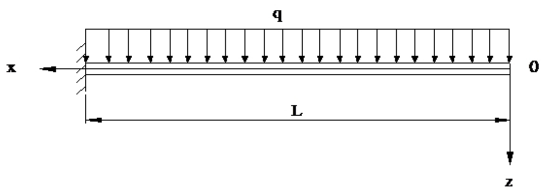

Figure 4. Cantilever beam bearing the uniformly distributed load.

The shear force and bending moment suffered by the section of cantilever beam are obtained easily: $M(x)=\frac{-1}{2}qx^2$ and $Q(x)=-qx$, which are applied in Eq. (16), the generalized displacement $u(x)$ is obtained.

$$u(x)=\frac{9nP}{8EI}\left(\frac{x}{k^2}+C_5\sinh(kx)+C_6\cosh(kx)\right). \quad (27)$$

The coefficient obtained is as follows with the equilibrium condition $u'\big|_{x=0}=0$ and $u'\big|_{x=l}=0$:

$$C_5=-\frac{1}{k^3},\quad C_6=-\frac{1}{\cosh(kl)}\left(l-\frac{1}{k}\sinh(kl)\right).$$

Thus, the shear lag coefficient obtained is as follows:

$$\lambda^e=1+\frac{9nI_s}{5x^2k^2I}\left[1-\cosh(kx)+\tanh(kl)-\frac{1}{\cosh(kl)}\sinh(kx)\right] \quad (28)$$

$$\lambda^c=1-(1-\frac{4}{5}\frac{I_s}{I})\frac{EI}{M_y}\cdot\frac{9n}{4x^2k^2}\left[1-\cosh(kx)+\left(\tanh(kl)-\frac{1}{\cosh(kl)}\sinh(kx)\right)\right] \quad (29)$$

3 Simulated analysis example

3.1 Computational model

The computational model is shown in Fig. 5; the width value (b) of the flange plate of the box girder is taken two under conditions respectively, i.e. 450 mm and 250 mm. The elasticity modulus of box girder materials is E=3.15·10^4 MPa and $G=0.4E$. The shear lag effect of the above mentioned box girder is analyzed with the finite element method and analytical computational method of the paper. The finite element computational model is shown in Fig. 6.

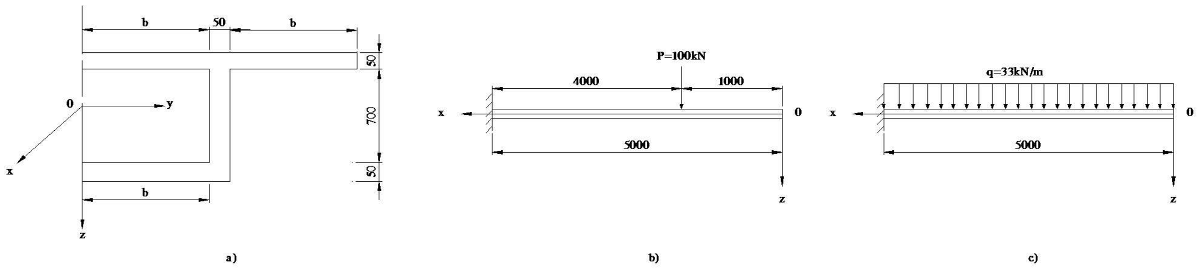

Figure 5. Computational model (Unit: mm): (a) Sectional dimension of box girder. (b) Cantilever beam bearing the concentrated load. (c) Cantilever beam bearing the uniformly distributed load.

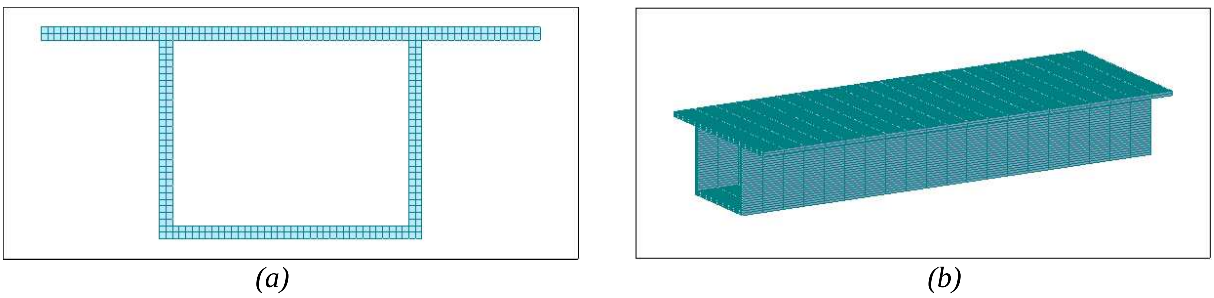

Figure 6. Finite element model: (a) Finite element model for the section of box girder. (b) Finite element model of cantilever beam.

3.2 Computed result

1. Cantilever beam bearing the concentrated load

When *b/l*=1/16.7 and *x* = 1250 mm, 2250 mm, 3750 mm, 4750 mm, the bending normal stress of the section of the upper flange plate of the box girder is computed as Fig. 7 to Fig. 10.
When *b/l*=1/10 and *x* = 1250 mm, 2250 mm, 3750 mm, 4750 mm, the bending normal stress of the section of the upper flange plate of the box girder is computed as Fig. 11 to Fig. 14.

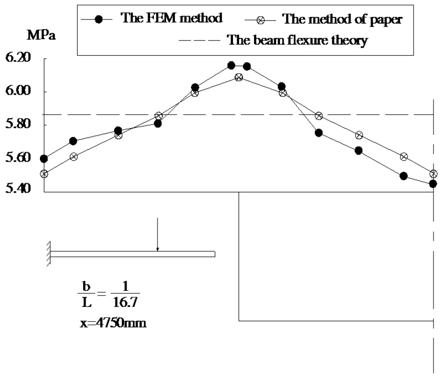

Figure 7. Stress of box girder at different sections (b/l=1/16.7 and x=4750 mm).

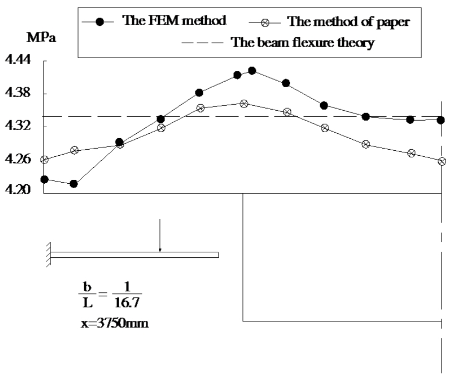

Figure 8. Stress of box girder at different sections (b/l=1/16.7 and x=3750 mm).

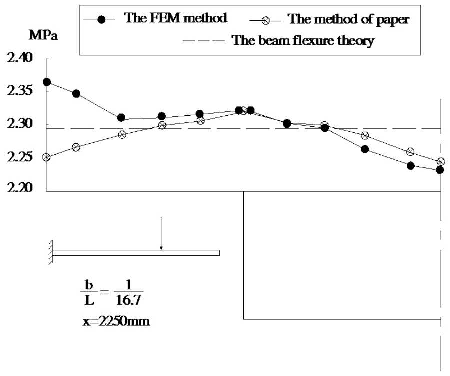

Figure 9. Stress of box girder at different sections (b/l=1/16.7 and x=2250 mm).

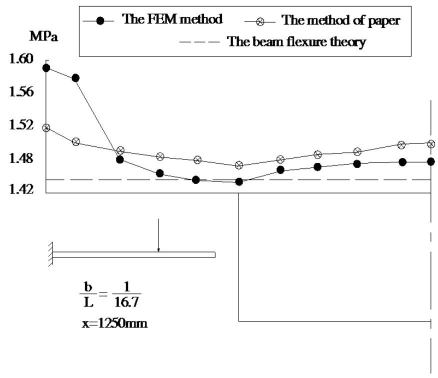

Figure 10. Stress of box girder at different sections (b/l=1/16.7 and x=1250 mm).

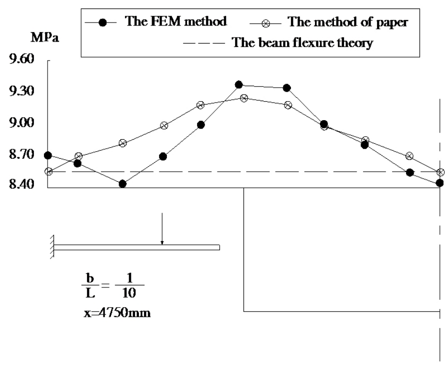

Figure 11. Stress of box girder at different sections (b/l=1/10 and x=4750 mm).

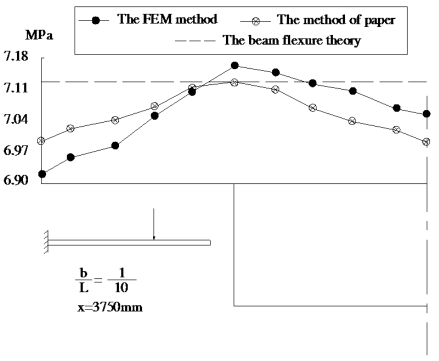

Figure 12. Stress of box girder at different sections (b/l=1/16.7 and x=3750 mm).

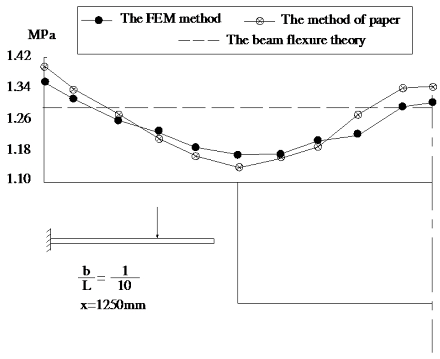

Figure 14. Stress of box girder at different sections (b/l=1/10 and x=2250 mm).

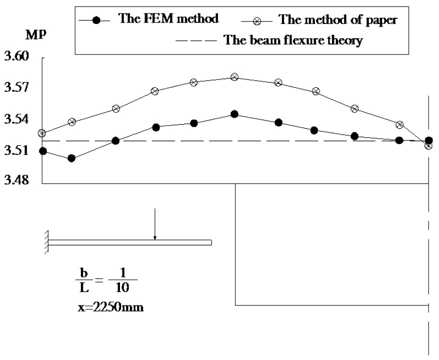

Figure 13. Stress of box girder at different sections (b/l=1/10 and x=2250 mm).

The computation of the shear lag coefficients at the different sections is shown in Table 1.

From Fig. 7 to Fig. 14 and Table 1, it is obtained that when the cantilever box girder is under the effect of the concentrated load, the section of box girder is occurred the shear lag effect, and there is the negative shear lag effect. The positive shear lag effect is remarkable at the place near the fixed end; the shear lag coefficient is reduced gradually with the increase of the distance, and is transited to the negative shear lag effect from the positive shear lag effect gradually. *b*/*l* is a parameter affecting the shear lag effect, and either the positive or negative shear lag is decreased equally with *b*/*l*.

Table 1. Computation table of shear lag coefficients (Cantilever beam bearing the concentrated load).

b/*l*	*x*(*mm*)	Finite element method		Method of the paper	
		λ^e	λ^c	λ^e	λ^c
$\frac{1}{16.7}$	4.75	1.068	0.915	1.034	0.933
	3.75	1.032	0.998	1.025	0.980
	2.25	1.022	1.005	1.022	1.001
	1.25	0.995	1.082	0.981	1.016
$\frac{1}{10}$	4.75	1.128	0.965	1.107	0.989
	3.75	1.109	0.980	1.052	0.982
	2.25	1.002	0.987	1.016	1.001
	1.25	0.853	1.082	0.862	1.062

2. Cantilever beam bearing the uniformly distributed load

When *b*/*l*=1/16.7 and x = 1250 mm, 4750 mm, the bending normal stress of the section of the upper flange plate of the box girder is computed as Fig. 15 to Fig. 16. When *b*/*l*=1/10 and x=1250 mm, 4750 mm, the bending normal stress of the section of the upper flange plate of the box girder is computed as Fig. 17 to Fig. 18. The computation of the shear lag coefficients at the different sections is shown in Table 2.

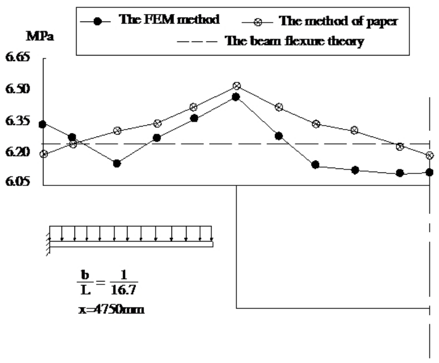

Figure 15. Stress of box girder at different sections (b/l=1/16.7 and x=4750 mm).

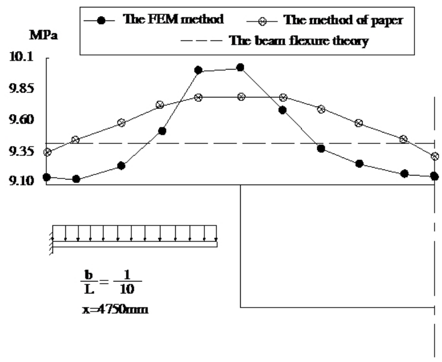

Figure 17. Stress of box girder at different sections (b/l=1/10 and x=4750 mm).

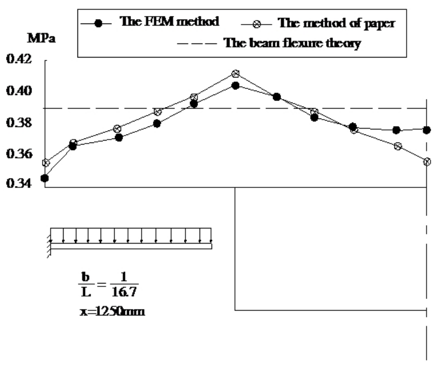

Figure 16. Stress of box girder at different sections (b/l=1/16.7 and x=1250 mm).

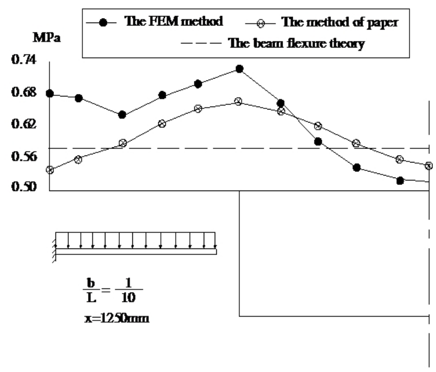

Figure 18. Stress of box girder at different sections (b/l=1/10 and x=1250 mm).

3.3 Computed result

From Fig. 15 to Fig. 18 and Table 2, it is obtained that when the cantilever box girder is under the effect of the uniformly distributed load, the section of box girder is occurred the shear lag effect, but there is no negative shear lag effect, which is consistent with the analytical result. In addition, in contrast to shear lag effect suffered from the concentrated load the shear lag effect is not obvious at the place near the fixed end, and the shear lag coefficient is remarkable gradually with the increase of the, distance. Also, *b*/*l* is a parameter affecting the shear lag effect, and either the positive or negative shear lag is decreased equally with the decrease of *b*/*l*.

From the finite element and theoretical analysis, it is obtained that the cantilever box girder structure will be occurred the shear lag phenomenon under the load effect and the nature of the shear lag effect is related to the form of applied load. Generally speaking, if the external load produces a constant shear flow within the section of cantilever beam, the positive shear lag effect will be produced only; and if the load produces a varying or reverse shear flow within the section of cantilever beam, the negative shear lag effect will be produced. The longitudinal displacement function $u(x,y)$, deciding the magnitude and nature of the shear lag, is composed of two parts, i.e. the influence of vertical displacement and the influence of maximum angle displacement difference function. To describe $u(x,y)$ correctly is crucial to the solution of shear lag. There are many factors affecting the magnitude of shear lag coefficient, in which the section parameters, including the width of the flange plate, web height and thickness, etc., will have influence on the magnitude of the shear lag coefficient. However, as a whole, for the width-span ratio (*b*/*l*) is a sensitive factor and the shear lag coefficient will be decreased remarkably with the decrease of width-span ratio.

Table 2. Computation table of shear lag coefficients (Cantilever beam bearing the uniformly distributed load).

b/l	$x(mm)$	Finite element method		Method of the paper	
		λ^e	λ^c	λ^e	λ^c
$\frac{1}{16.7}$	4.75	1.041	0.926	1.057	0.994
	1.75	1.051	0.981	1.071	0.923
$\frac{1}{10}$	4.75	1.078	0.995	1.043	0.970
	1.75	1.276	0.897	1.155	0.931

4 Conclusion

Based on exerting the principle of minimum potential energy and combining the variation method, the shear lag coefficient, additional bending moment and deflection are analyzed in detail under condition that the cantilever box girder is acted on the concentrated force load and the uniformly distributed load. The effectiveness and accuracy of the method are verified. The main conclusions made in this study are as follows:

1. The section of cantilever box girder will produce the shear lag effect after suffering the load, and if the external load produces a constant shear flow within the section of cantilever beam, the positive shear lag effect will be produced only; and if the load produces varying or reverse shear flow within the section of cantilever beam, the negative shear lag effect will be produced.
2. The shear lag coefficient reflects the non-uniform degree of section stress; the strength of section will be underestimated if only accorded with the elementary beam theory without regard to the shear lag effect, therefore, the potential safety hazard will be produced.
3. The shear lag coefficient of the cantilever box girder suffered from the concentrated load varies from big to small from the fixed end to the free end; the variation trend of the shear lag coefficient of the cantilever box girder suffered from the uniformly distributed load is inverse.
4. Either the positive or negative shear lag effect will produce the additional bending moment, which will increase the structural deflection.
5. The section characteristics will affect the shear lag coefficient, and for the width-span ratio (b/l) is a sensitive factor, the shear lag coefficient will be decreased remarkably with the decrease of the width-span ratio.
6. The conclusions above are only applied to the uniform cantilever beam, and further studies are required for the shear lag effect of the non-uniform cantilever beam.

References

[1] Yamaguchi, E., Chaisomphob, T., Lertsima, C., Sa-nguanmanasak, J.: *Stress concentration and deflection of simply supported box girder including shear lag effect*, Structural Engineering and Mechanics, 28 (2008), 2, 207-220.

[2] Štimac, G., Turkalj, G., Žigulić, R.: *Dynamic analysis of a cantilever column subjected to compressive force,* Engineering Review, 28 (2001), 1, 1-10.

[3] Sa-nguanmanasak, J., Chaisomphob, T., Yamaguchi, E.: *Stress concentration due to shear lag in continuous box girders*, Engineering Structures, 29 (2007), 7, 1414-1421.

[4] Chang, S.T.: *Shear lag effect in simply supported prestressed concrete box girder*, Journal of Bridge Engineering, 9 (2004), 2, 178-184.

[5] Luo, Q.Z., Tang, J., Li, Q.S.: *Negative shear lag effect in box girders with varying depth*, Journal of Structural Engineering, 127 (2001), 10, 1236-1239.

[6] Lertsima, C., Chaisomphob, T., Yamaguchi, E., Sa-nguanmanasak, J.: *Deflection of simply supported box girder including effect of shear lag,*. Computers & Structures, 84 (2005), 1-2, 11-18.

[7] Kumar, M.: *Influence of box-geometry on shear lag In RC box-girder bridges*, Indian Concrete Journal, 84 (2010), 1, 15-20.

[8] Wu, Y., Liu, S., Zhu, Y., Lai, Y.: *Matrix analysis of shear lag and shear deformation in thin-walled box beams*, Journal of Engineering Mechanics, 129 (2003), 8, 944-950.

[9] Zhou, M.D., Liu, S.Z., Yang, Z.J.: *Bar simulation method on shear lag effect of composite box girder with corrugated steel web*, Journal of Lanzhou Jiaotong University, 31 (2012), 4, 41–44.

[10] Li, F.X., Nie, J,G.: *Elastic Analytical Solutions OF Shear Lag Effect OF Steel-concrete Composite Beam*, Engineering Mechanics, 28 (2011), 9, 1–8.

[11] Reissner, E.: *Analysis of shear lag in box beams by the principle of the minimum potential energy*, Quarterly of Applied Mathematics, 4 (1946), 3, 268-278.

[12] Zhang, Y.H., Bai, X., Lin, L.X.: *An improved approach for analyzing shear lag effect of box girders*, China Civil Engineering Journal, 45 (2012), 11, 153–158.

[13] Zhang, Y.H.: *Improved finite-segment method for analyzing shear lag Effect in thin-walled box girder*, Journal Of Structural Engineering, 138 (2011), 10, 1279–1284.

[14] Cao, G.H., Fang, Z.: *Shear lag effect of cracked simply supported box girder analyzed by variational principle*, Chinese Journal of Computational Mechanics, 24 (2007), 6, 853–864.

[15] Cao, G.H., Fang, Z.: *Shear lag effect of cracked continuous box girder analyzed by variational principle*, Engineering Mechanics, 24, 4, 75–80, 2007.

[16] Li, Q.F., Lu, B.W., Zhou, M.D.: *Shear lag effect of box girder with varying loading positions*, Journal of Zhengzhou University, 32 (2011), 2, 59-61.

[17] Zhou, Y.C., Sun, T.J., Hao, X.W.: *Analysis of Shear Lag Effect Based on The Shear Flow Distribution Pattern*, Earthquake Resistant Engineering And Retrofitting, 35 (2013), 2, 44–49.

[18] Zhou, S.J.: *Shear lag analysis in prestressed concrete box girders*, American Society of Civil Engineers, 16 (2011), 4, 500–512.

[19] Zhou, S.J.: *Shear lag analysis of box girders*, Engineering Mechanics, 25 (2008), 2, 204-208.

[20] Singh, Y., Nagpal, A.K.: *Negative shear lag in framed-tube buildings. Journal of Structural Engineering*, Journal of Structural Engineering, 120 (1994), 11, 3105-3121.

[21] Shushkewich, K.W.: *Negative shear lag explained*, Structural Engineering, 117 (1997), 3543-3546.

[22] Kroiostek, V., Studnicoka, J.: *Negative shear lag in flanges of plated structures*, Journal of Structural Engineering, 117 (1997), 12, 3553-3569.