GLUED TIMBER-CONCRETE BEAMS – ANALYTICAL AND NUMERICAL MODELS FOR ASSESSMENT OF COMPOSITE ACTION

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Abstract:

An analysis of glued composite timber-concrete systems is presented. Experimental data obtained from laboratory tests under short-term loading are compared with the analytical calculation and the design procedure for fully composite beams given in the EN 1995-1-1 standard. Numerical linear 2D finite element modelling and an analytical solution assuming linear elastic behaviour of glue and the interlayer slip are also conducted and validated. The effect of composite action in the three mentioned approaches is assessed by comparison of midspan deflections. In this way, a parametric study of the glue-line properties and the interlayer slip stiffness on load-carrying capacity and serviceability of glued composite beams exposed to short-time loading is easily performed.

1 Introduction

Composite timber-concrete systems offer several applications in structural engineering. Although this technology is well known and has been investigated for over fifty years, mainly for reconstruction works and reinforcement of solid timber floors, nowadays research is mostly focused on possibilities to use these systems in new structures, multi-storey buildings and bridges [1]. With concrete working in compression and timber working in tension, they make the most of each material, resulting in an improved global behaviour. This behaviour will mainly depend on the interface connection between the two materials. The glued connection enables the best interaction between the two materials and prevents slippage at the interface, which occurs when mechanical connectors such as nails, spikes, bolts and dowels are used [2]. In the present work the midspan deflections obtained from laboratory flexural tests of a glued timber-lightweight EPS concrete specimen [3] have been compared with the results obtained from the analysis according to EN 1995-1-1 [4], as well as the results using the finite element method and an analytical solution for the composite two-layer beam [5]. The behavior of glued timber-concrete composites is a very interesting field of research primarily because of its potential of wide application in the structural design and pre-cast assemblies. It certainly requires investigation of phenomena specific to these systems, which come as a result of the long-term loading effects, and the effects of the type of glue used and the bonding technique applied on strength and stiffness of the composite beam [6]. The effects of variations in environmental humidity and moisture content (especially in timber) as well as the dimensional variations (e.g. when concrete

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shrinks) are particularly interesting due to the rheological material properties (e.g. creep) [6] together with the effects of temperature change in the concrete part. These phenomena may become very significant for stiff connections and therefore require additional consideration and investigation. Such comprehensive investigations become even more interesting and necessary when the light-weight concrete with smaller Young's modulus and a bigger expected creep is used instead of standard concrete. The target field of the application of the timber - light-weight concrete composites are the floor systems in buildings, where a consideration of the composite action and the changes of Young's modulus are required. Since no experimental data on long-term loading behavior [2] were available to the authors to compare their results with, the present work deals only with behavior of the glued timber-light-weight concrete composites under a short-term loading. The main aim of the authors is to test the validity of the numerical and analytical methods used, and, on the basis of comparison of the corresponding results with the experimental data [2], to evaluate the suitability of the presented approach for parametric studies of such composites, with the focus on the development of the analytical solution given in Section 5.

2 Experimental data

The tested specimen is a simply supported beam with a T-shaped cross-section assembled from a glued laminated (GluLam) timber beam and a lightweight expanded polystyrene (EPS) concrete slab (see Figs. 1 and 2). The two parts are continuously glued together using two-component epoxy resin (EPOCON 88) applied in a 2 mm thick layer. The mechanical properties (compressive, tensile and shear strength, modulus of elasticity) of the materials used were determined by destructive testing [2]. For the purpose of this research, only the mean values of moduli of elasticity are relevant and they read $E_{c,m}$= 12400 N/mm^2 (EPS concrete), $E_{0,mean}$ = 12800 N/mm^2 (GluLam timber) and E = 7066 N/mm^2 (Epoxy resin). The beam is loaded by a concentrated force applied at the midspan via a steel plate (20 mm thick, 100 mm long and 400 mm wide), while two additional steel platens 20 mm thick and 100 mm long have been placed at the supports (Fig. 1). The corresponding deflections are measured by the five LVDTs (from a total of fifteen) placed at the bottom edge of the timber beam at the midspan, the other ten being placed at the bearings and at the ends of the EPS slab. The relationship between the applied force and the midspan deflection is shown in Fig. 3.

The force until failure is obtained as P_F=154 kN and it can be noticed that the actual collapse force (approximately 160 kN) is very near (see Fig. 3). For further analysis, a representative load of the value $Q=P_F/4$=38.5 kN is chosen with its corresponding deflection, roughly assessed as w_m = 6.0 mm. From Fig. 3 it is obvious that for such a representative load the actual behavior of the composite structure is well below the limit of proportionality, which enables assessing its stiffnes

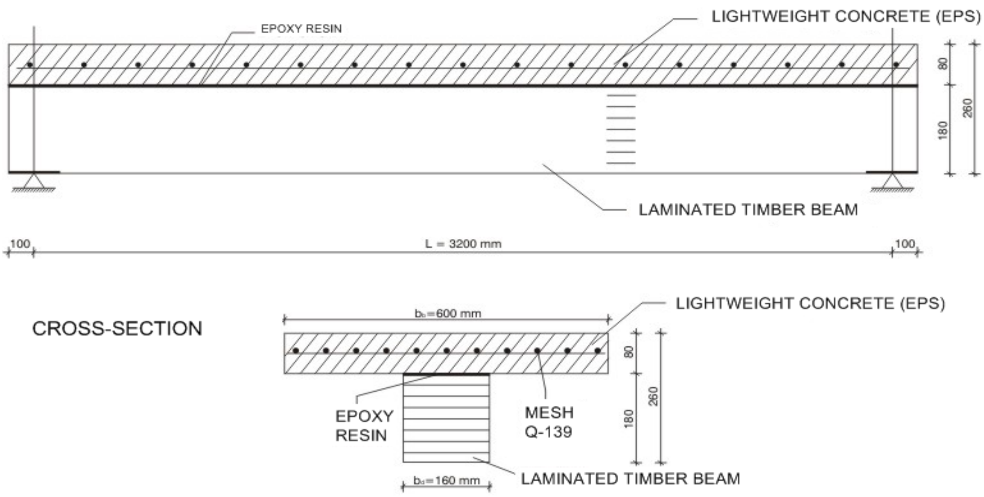

Figure 1. Tested composite timber-concrete beam specimen [2].

Figure 2. Setup of the described experiment [2].

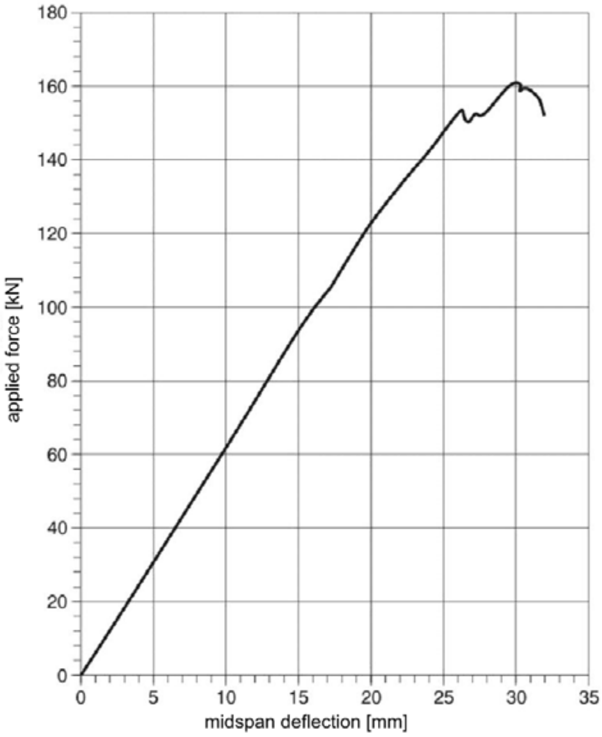

Figure 3. Applied load vs. midspan deflection relationship [2].

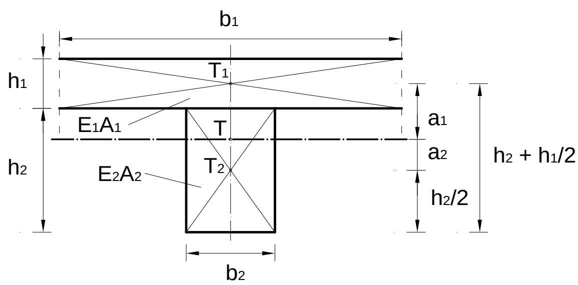

Figure 4. Eccentricities of the layer's axis of centroids with respect to the axis of centroids of the fully composite beam.

as an approximately constant value. Unfortunately, the deflections measured at the supports are not known and therefore they are not considered in the present analysis. The experimental data for the slip at the edges of the beam and along the length of the beam were not available, nor were the data from the small specimen shear tests. This is a factor which must be considered when assessing the correspondence of the experimental data with the results obtained from the presented numerical and analytical methods.

3 Analysis of glued timber-concrete composite beams according to Eurocode5

Although Part 2 of standard EN 1995 deals with timber-concrete composite members, the design procedure relevant for the glued composites is given in Part 1-1 of EN 1995. According to section 9 in EN 1995-1-1 [4], the design procedure is based on the assumption that the glued connection is absolutely rigid, thus no slip at the interface occurs. In line with the EN 1995-1-1 standard, the flexural design stiffness $(EI)_{ef}$ of a glued beam with an ideal bond is defined as

$$(EI)_{ef} = (EI)_{ef,id} = \sum_{i=1}^{2}\left(E_i I_i + E_i A_i a_i^2\right), \qquad (1)$$

where, for each layer i of the section, E_i, A_i and I_i are Young's modulus, the cross-sectional area and its second moment with respect to its own axis of centroids, while a_i is eccentricity of the layer's axis of centroids with respect to the axis of centroids of the fully composite beam. Here i=1 for the concrete slab and i=2 for the glued laminated timber beam.
For the cross-section analysed (see Fig. 4), these eccentricities follow as

$$a_1 = \frac{E_2 A_2 \left(h_1 + h_2\right)}{2\left(E_1 A_1 + E_2 A_2\right)} = 49.72\,\text{mm}\,, \qquad (2)$$

$$a_2 = \frac{h_1 + h_2}{2} - a_1 = 80.28\,\text{mm}\,, \qquad (3)$$

while the design stiffness $(EI)_{ef}$ follows from (1) as $(EI)_{ef}$= 5.16·10^{12}N/mm^2. Considering the deflection due to selfweight in addition to the deflection due to the concentrated force Q=P/4 at the midspan we obtain

$$w_{id} = \frac{5ql^4}{384(EI)_{ef}} + \frac{Ql^3}{48(EI)_{ef}} = 5.30\,\text{mm}\,. \qquad (4)$$

The selfweight q was calculated from the cross-sectional areas of the layers (see Fig. 1) and their associated densities: 410 kg/m^3 for GluLam timber and 1400 kg/m^3 for EPS slab. In this case, the calculated deflection is approximately 12% smaller than the deflection measured in the test, and the design procedure according to EN 1995-1-1 represents a rough idealization of the real behaviour of the glued composite beams. The calculated deflection is therefore underestimated and it obviously represents the upper limit for the effective bending stiffness of composite beam when slippage along the contact line does not occur at all (5). The lower limit of the effective bending stiffness is represented when there is no connection between the layers and they act as unbounded. The stiffness of such a composed beam is defined as

$$EI_0 = E_1 I_1 + E_2 I_2\,, \qquad (5)$$

and the midspan deflection in this case is

$$w_0 = \frac{5ql^4}{384EI_0} + \frac{Ql^3}{48EI_0} = 20.84\,\text{mm}\,. \qquad (6)$$

Thus, depending on a measure of bonding stiffness, the composite beam deflections will always vary between limits (4) and (6). For the present case it is obvious that the behaviour of the glued composite beam is very close to the behaviour of the fully composite beam. Note that this procedure does not take into account influence of shear deformation on the deflection.

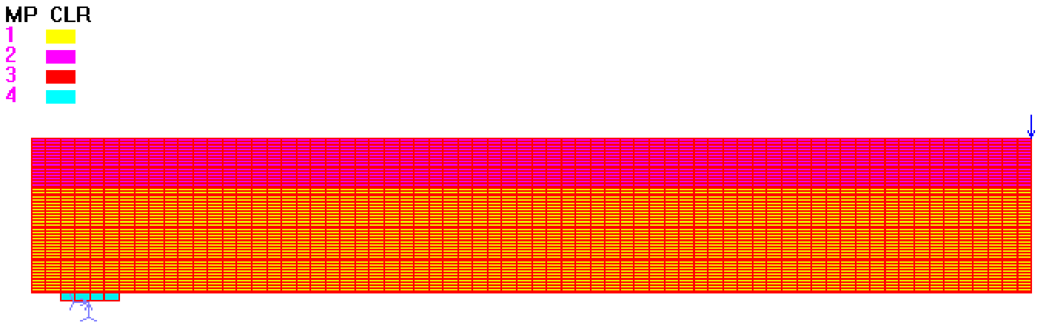

Figure 5. Finite-element mesh.

4 An analysis using finite element method

A plane finite element analysis for the tested composite timber-EPS beam is performed using the computer package "COSMOS/M". The four node isoparametric plane stress PLANE2D finite elements are used, where only two translational degrees of freedom exist at a node.

Only the isotropic material properties are considered for timber part of the composite beam with the following values: E_x = 12800 N/mm^2 and G_{xy} = 650 N/mm^2 and a density of 410 kg/m^3. Modulus of elasticity E_x is the mean value determined by tests while the shear modulus G_{xy} is approximated in accordance with the European norm EN 408 [7]. Appropriate material properties for the EPS-part are: E_x = 12400 N/mm^2 and G_{xy} = 4960 N/mm^{2}and a density of 1400 kg/m^3. The thin epoxy resin layer is also modelled, thus the interface connection is considered as not completely rigid, which illustrates the real behaviour. The material properties of the epoxy resin are taken as E_x = 7066 N/mm^2, G_{xy} = 2644.5 N/mm^2 and density 1200 kg/m^3. Additionally, the steel platens at the midspan and the supports have been also modelled using the finite elements of the same type with E_x = 21000 N/mm^2 and G_{xy} = 8100 N/mm^2 and with density of 7850 kg/m^3. Due to symmetry, only one half of the beam has been analysed with the horizontal displacements at the symmetry line constrained. The EPS layer has been modelled using 16 x136 elements (total of 2176 FE), the epoxy resin layer has been modelled using 1 x 136 elements and the GluLam layer has been modelled using 36 x 136 elements (total of 4896 FE). 1 x 4 elements have been used for each steel platen at the support. The vertical displacement of the central node at the bottom of the platen at the support has been constrained. The finite-element mesh is shown in Fig. 5. Such a model has been loaded with the selfweight and a concentrated vertical force of 38.5 kN acting at the top node along the symmetry line. The finite element analysis conducted in this way results in w (L/2)=5.84 mm as the maximum deflection of the bottom node at the midspan of the beam. The deformed shape of the composite beam is shown in Fig. 6.

5 Analytical solution of a two-layer composite beam

A linear elastic analytical model for a composite two-layer beam using the Euler-Bernoulli beam theory is proposed in [5] and is adopted here for comparison purposes. In addition to the results given in [5], we derive a particular solution for a uniformly distributed loading. We also pay special attention to proving that the limiting values of the solution in the situations when the bond is fully rigid or the layers act as completely separate are the expected engineering results. The model allows only for the interlayer slip, while the interlayer uplift is not considered. Thus, we denote the displacements of the layers with respect to their axes of centroids as u_1 and u_2 in the x-direction, and w_1=w_2=w in the z-direction. The friction between the layers is negligible and the stiffness of the connection is defined by a constant K [N/mm^2], called the slip modulus. The interlayer traction is then defined as

$$V_s = K \cdot \Delta u = K \cdot \left(u_2 - u_1 + \frac{dw}{dx} \cdot r \right), \qquad (7)$$

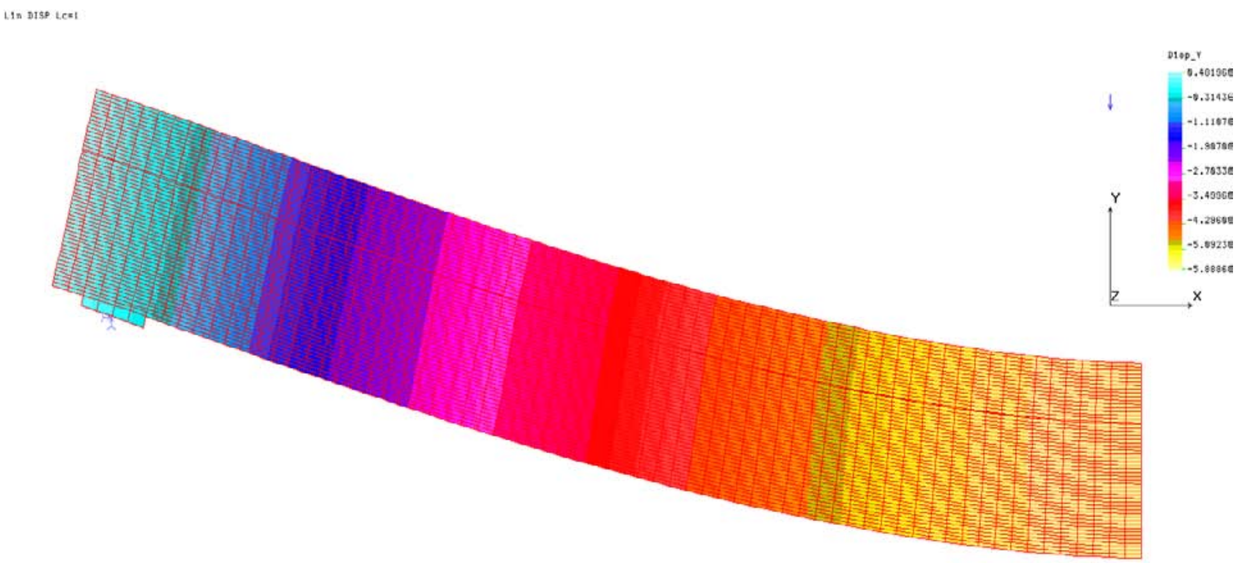

Figure 6. A finite element model for the composite timber-EPS specimen.

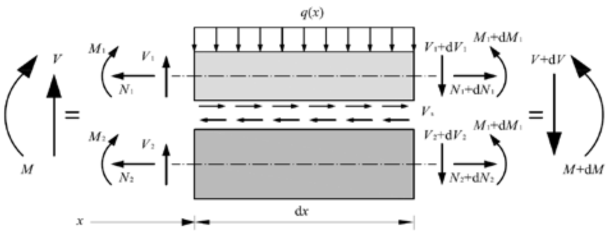

Figure 7. Equilibrium of a two-layer beam section.

where Δu represents the interlayer slip and r is the distance between the axes of centroids of the layers (see Fig. 7).

The bending stiffness in case with no interlayer connection (K=0) is defined as $EI_0 = E_1 I_1 + E_2 I_2$. In case of a fully composite beam with K=∞, the bending stiffness can be expressed [4] as

$$EI_\infty = \frac{EI_0}{1 - \beta \cdot r / \alpha^2}, \tag{8}$$

where

$$\alpha^2 = K \cdot \left(\frac{1}{E_1 A_1} + \frac{1}{E_2 A_2} + \frac{r^2}{EI_0} \right) \text{ and } \quad \beta = \frac{K \cdot r}{EI_0}. \tag{9}$$

These constants are introduced to simplify the basic equations of the problem and isolate the influence of the slip modulus K, but when they are substituted into expression (8) we obtain

$$EI_\infty = EI_0 + \frac{E_1 A_1 E_2 A_2 r^2}{E_1 A_1 + E_2 A_2}. \tag{10}$$

It can be shown that $EI_\infty = (EI)_{ef.id}$ [3]. The equilibrium equations are written according to Fig.7 as

$$N_1 = -N_2,$$
$$M = M_1 + M_2 - N_1 \cdot r,$$
$$V_s = -\frac{dN_1}{dx} = \frac{dN_2}{dx}, \tag{11}$$
$$V_1 + V_2 = \frac{dM_1}{dx} + V_s \cdot r_1 + \frac{dM_2}{dx} + V_s \cdot r_2.$$

Using these relations and considering the kinematic and constitutive relations from the Euler-Bernoulli beam theory

$$\frac{d^2w}{dx^2} = -\frac{M_i}{E_i I_i}, \quad \frac{du}{dx} = \frac{N_i}{E_i A_i}, \tag{12}$$

where i=1,2, we arrive at a sixth order differential equation for $w(x)$:

$$\frac{d^6w}{dx^6} - \alpha^2 \cdot \frac{d^4w}{dx^4} = -\frac{\alpha^2}{EI_\infty} \cdot q + \frac{1}{EI_0} \cdot \frac{d^2q}{dx^2}. \tag{13}$$

The solution in case of constant external loading $q(x)$=const. is given in the form [2]

$$\begin{aligned} w(x) &= a_1 \sinh(\alpha x) + a_2 \cosh(\alpha x) \\ &+ a_3 x^3 + a_4 x^2 + a_5 x + a_6 + \frac{q \cdot x^4}{24 \cdot EI_\infty}, \end{aligned} \tag{14}$$

where the constants $a_1,\ldots,a_6$ are obtained from the boundary conditions. See [3] for the detail. In our case of a simply supported beam with a concentrated misdspan load Q and the selfweight defined as a uniformly distributed load q, the boundary conditions read

$$w = 0, \; M = 0, \; V = \frac{Q}{2} + \frac{q \cdot L}{2}, \; N_1 = 0, \tag{15}$$

for x=0, and

$$\frac{dw}{dx} = 0, \; M = \frac{Q \cdot L}{4} + \frac{q \cdot L^2}{8}, \; V = \frac{Q}{2}, \; \Delta u = 0, \tag{16}$$

for $x=L/2$. The result for the midspan deflection w(L/2) follows as

$$\begin{aligned} w(L/2) &= \frac{Q \cdot L^3}{48 \cdot EI_\infty} \cdot \left[1 + 12 \cdot \left(\frac{EI_\infty}{EI_0} - 1\right) \cdot c_1\right] + \\ &+ \frac{5 \cdot q \cdot L^4}{384 \cdot EI_\infty} \cdot \left[1 + \frac{48}{5} \cdot \left(\frac{EI_\infty}{EI_0} - 1\right) \cdot c_2\right], \end{aligned} \tag{17}$$

where

$$\begin{aligned} c_1 &= \frac{\alpha L - 2 \cdot \tanh(\alpha L/2)}{(\alpha L)^3}, \\ c_2 &= \frac{8 \cdot \left[\cosh^{-1}(\alpha L/2) - 1\right] + (\alpha L)^2}{(\alpha L)^4}. \end{aligned} \tag{18}$$

The coefficients c_1 and c_2 depend on the slip modulus K via α and have the following limits

$$\begin{aligned} &\lim_{\alpha L \to 0} \frac{\alpha L - 2 \cdot \tanh(\alpha L/2)}{(\alpha L)^3} = \frac{1}{12}, \\ &\lim_{\alpha L \to \infty} \frac{\alpha L - 2 \cdot \tanh(\alpha L/2)}{(\alpha L)^3} = 0, \end{aligned} \tag{19}$$

$$\begin{aligned} &\lim_{\frac{\alpha L}{2} \to 0} \frac{8 \cdot \left[\cosh^{-1}(\alpha L/2) - 1\right] + (\alpha L)^2}{(\alpha L)^4} = \frac{5}{48}, \\ &\lim_{\frac{\alpha L}{2} \to \infty} \frac{8 \cdot \left[\cosh^{-1}(\alpha L/2) - 1\right] + (\alpha L)^2}{(\alpha L)^4} = 0. \end{aligned} \tag{20}$$

leading to the standard engineering results for a dual layer beam with no interlayer connection or the full interlayer connection

$$\begin{aligned} &\lim_{\alpha L \to 0} w(L/2) = \frac{Q \cdot L^3}{48 \cdot EI_0} + \frac{5 \cdot q \cdot L^4}{384 \cdot EI_0}, \\ &\lim_{\alpha L \to \infty} w(L/2) = \frac{Q \cdot L^3}{48 \cdot EI_\infty} + \frac{5 \cdot q \cdot L^4}{384 \cdot EI_\infty}. \end{aligned} \tag{21}$$

In case of the material and geometrical properties of the tested specimen (see chapters 2 and 3), we obtain the following result for $Q=P/4$

Table 1. A comparison of results for midspan deflection.

Source	Deflection (mm)
Laboratory test	6.00
EN 1995-1-1, fully composite	5.30
EN 1995-1-1, non-composite	20.84
FEM model	5.84
Analytical model	5.65

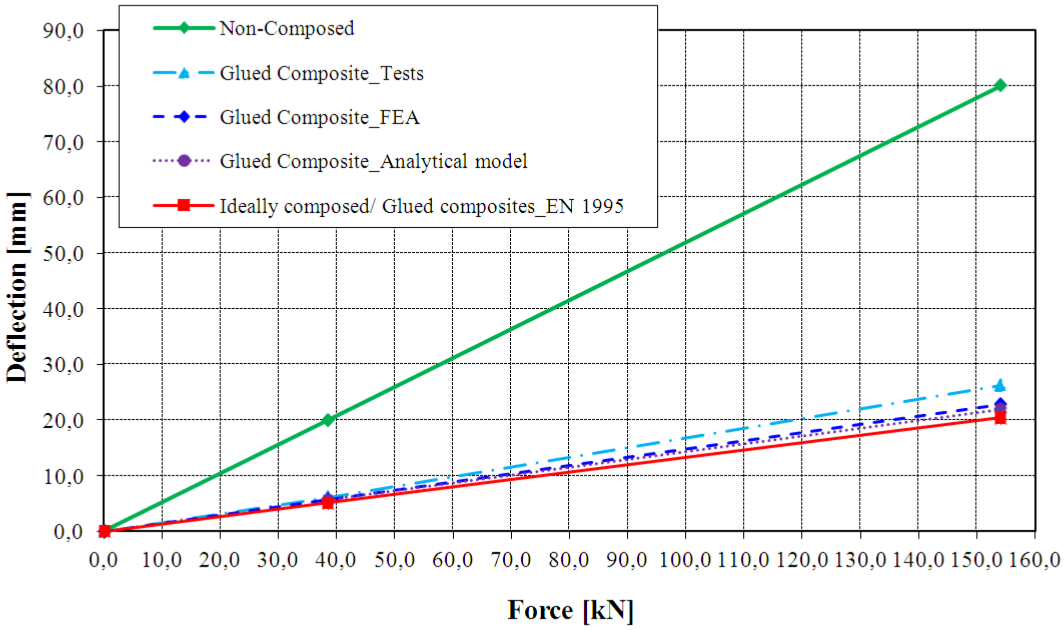

Figure 8. A comparison of the results obtained by different analyses (force-deflection diagram).

$$w_a = \frac{Q \cdot L^3}{48EI_\infty} \cdot 1{,}068 + \frac{5}{384} \cdot \frac{q \cdot L^4}{EI_\infty} \cdot 1{,}059 = \qquad (22)$$
$$= 5.65\,\mathrm{mm}.$$

The value of the slip modulus K, which is necessary to determine the constants α^2 and β, was taken as the value of the shear modulus of the epoxy glue (K=G=2644.5 N/mm^2). Comparing expressions (4) and (22), it can be noticed that the deflections from the analytical model considering the interlayer slip are larger than the deflections for the fully composite beam. The deflections from the concentrated force and the selfweight in expression (17) are magnified by the factors from (18) which depend on the material and geometrical properties of the beam, but also on the slip modulus K, which is contained within constant α. Note that this analytical model also gives the slip between the two layers as well as the internal forces and moments, which, however, are not the subject of the present comparison. See [5] for further detail.

6 Comparison of the results

The results obtained from the laboratory tests (Section 2) with the results obtained using the procedure according to EN 1995-1-1 (Section 3), the finite element model (Section 4) and the analytical model (Section 5) are summarised in Table 1. In Fig. 8 the same comparison is shown for values of the load 0.25P and P.

From Table 1 and Fig. 6, it can be noticed that the measured midspan deflections are larger than the deflections calculated from the numerical and analytical models, assuming interlayer slip and from the analysis according to EN-1995-1-1, assuming fully composite beam. This is partially due to not taking into account the shear deformations in our analytical model, but also due to material and geometrical imperfections. Also, in both the numerical and the analytical model the GluLam beam has been treated as isotropic, which is only an idealization of the real orthotropic behavior.

7 Conclusions

In the present work it has been shown that the assumption of an absolutely rigid interface connection of glued composite beams proposed by EN-1995-1-1 does not describe the real nature of the problem. It has been demonstrated that the real deflections of the glued timber-concrete beam are larger than those calculated according to the EN-1995-1-1 procedure. This simplification, however, is not on the safe side of the design process. Rather simple numerical and analytical models are in a very good agreement with the laboratory tests. Numerical modelling using the finite element method is always recommended for the analysis of such systems, but the results from the analytical model are sufficiently accurate and can be easily applied in engineering practice. To gain more accuracy, for the analytical model, shear deformations can be taken into account.

Further research assisted by relevant testing, which should take into account long-time behaviour of the composed beam is needed to gain additional insight into this complex behaviour.

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