INFLUENCE OF THE FIBRE DAMPING COMPUTATIONAL MODEL IN A MECHANICAL SYSTEM ON THE COINCIDENCE WITH THE EXPERIMENTAL MEASUREMENT RESULTS

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ARTICLE INFO	Abstract:
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Keywords:	over a pulley. At its other end there is a prism-
Fibre	shaped steel weight which moves in a prismatic
Mechanical system	linkage on an inclined plane. In the presented case,
Dynamic response	the position of the weight is symmetric with respect
Phenomenological model	to the plane of drive-pulley symmetry. Drive
	periodic excitation signals can be of different
	shapes with the possibility of variation of a signal
	rate. Time histories of the weight position and the
	force acting in the fibre are measured. The same
	system is numerically investigated by means of
	multibody models. In the computational model, the
	influence of the fibre damping coefficient exerted
	both on the coincidence of the simulation results
	and the experimental measurement results is
	evaluated. The main aim of simulations is to create
	a phenomenological model of a fibre which will be
	utilizable in fibre modelling in the case of more
	complicated mechanical or mechatronic systems.

1 Introduction

The replacement of the chosen rigid elements of manipulators or mechanisms by fibres or cables [1] is advantageous due to the achievement of a lower moving inertia, which can lead to a higher machine speed and lower production costs. Drawbacks of using the flexible elements like that can be associated with the fact that cables should be only in tension (e.g. [2, 3]) in the course of motion. Experimental measurements focused on the investigation of the fibre behaviour are performed

on an assembled weight-fibre-pulley-drive (WFPD)

system [4-8]. The fibre is driven with one drive, it is led over a pulley and on its other end there is a prism-shaped steel weight, which moves on an inclined plane. The position of the weight can be symmetric (see Fig. 1 and Fig. 2) or asymmetric with respect to the plane of a drive-pulley symmetry (note: results with symmetric position of the weight are presented in this paper). It is possible to add an extra mass to the weight. Time histories of the weight position and of the force acting in the fibre are measured. The same system is numerically investigated using multibody models created in the *alaska* simulation tool [9]. Computational models

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(e.g. [10-12]), in this case multibody models of WFPD mechanical system were utilized for the investigation of dynamic behaviour of real systems (e.g. [13-15]). The influence of the model parameters exerted both on the coincidence of the results of experimental measurements and the simulations results is evaluated. The simulation aim is to create a phenomenological model of a fibre, which will be utilizable in fibre modelling in the complicated mechanical case of more or mechatronic systems.

The fibre damping coefficient, the fibre stiffness and the friction force acting between the weight and prismatic linkage were considered to be system parameters of the phenomenological model. The parameters determined for investigating the weightfibre system [13] were applied to the fibre model of the WFPD system. The friction force acting between the weight and prismatic linkage, as it has been confirmed [4-6], is not the parameter of the phenomenological model. This quantity is dependent on the angle α of the inclined plane (see When simulating the experimental Fig. 2). measurements for "slower" drive motion [4-6], the local extremes of the time histories of the weight displacement and of the force acting in the fibre are independent of the fibre stiffness and the fibre damping coefficient (considered in feasible intervals of values). When simulating the experimental measurements for "quicker" drive motion [4-6], the local extremes of the monitored time histories are dependent on both phenomenological model parameters, namely, frequencies of drive motion, i.e., frequencies of a periodic input signal higher than 1 Hz are designated as "quicker" drive motions, whereas frequencies of a periodic drive motion lower than 1 Hz are designated as "slower" drive motions. From the obtained results it was evident that these parameters of the fibre phenomenological model must be, in addition, considered dependent on the velocity of the weight motion [7, 8]. That is why the influence of the velocity-dependent damping "coefficient" considered in the fibre model and exerted on dynamic response of the system is investigated in this paper.

Results of both experimental measurements and simulations of five selected tested situations (altogether 43 situations were tested) at weight symmetric positions are presented in this paper (see Figs. 4 to 15). Four tested situations are at a

"quicker" drive motion (see time histories of drive motion in Fig. 4, Fig. 6, Fig. 8 and Fig. 10) and one situation is at a "slower" drive motion (see time history of drive motion in Fig. 13). Parameters of these tested situations are specified in Table 1.



Figure 1. Real WFPD mechanical system.



Figure 2. Scheme of a WFPD mechanical system.

Tested situation	Shape of input signal	Frequency of input signal [Hz]	Added mass	Related figures
2	trapezoidal	1.25	no	Fig. 4, Fig. 5
3 c	trapezoidal	2	no	Fig. 6, Fig. 7
7 a	trapezoidal	1.25	yes	Fig. 8, Fig. 9
10	trapezoidal	2	yes	Fig. 10, Fig. 11, Fig. 12
11	quasi-sinusoidal	0.35	no	Fig. 13, Fig. 14, Fig. 15

Table 1. Parameters of presented tested situations

2 Experimental stand

focused Experimental measurements on the investigation of the fibre behaviour are performed in an assembled WFPD mechanical system (see Fig. 1). A carbon fibre with a silicone coating (see e.g. [13]) is driven with one drive and led over a pulley. The fibre length is 1.82 meters (fibre weight is 4.95 grams), the pulley diameter is 80 millimetres. The weight position can be symmetric [5] (see Fig. 1 and Fig. 2) or asymmetric [6] with respect to the vertical plane of drive-pulley symmetry (the distance of the weight from the vertical plane of drive-pulley symmetry is 280 millimetres in the case of the asymmetric weight position). The fibre driven with one drive is fixed on a force gauge. At the other end of the fibre there is a prism-shaped steel weight (weight 3.096 kilograms) moving in a prismatic linkage on an inclined plane. It is possible to add an extra mass (weight 5.035 kilograms) to the weight [5]. The angle of inclination of the inclined plane can be changed. In the case of the symmetric weight position, the angle is $\alpha = 30$ degrees and the pulleyfibre angle is $\varphi = 150$ degrees (in the case of the asymmetric weight position, the angle is $\alpha = 30.6$ degrees and the pulley-fibre angle is $\varphi = 146$ degrees). Drive periodic exciting signals can be of a rectangular, a trapezoidal and a quasi-sinusoidal shape and there is a possibility of variation of a signal rate [15]. The amplitudes of the drive displacements x are up to 90 millimetres. Time histories of weight position *u*, in the direction of the inclined plane and measured by means of a dial gauge, of drive position x (in vertical direction) and of the force acting in the fibre measured by a force gauge at drive are recorded using a sample rate of 2 kHz.

3 Possibilities of the fibre modelling

The fibre (cable, wire etc.) modelling [16] should be based on considering fibre flexibility and suitable approaches can be based on the flexible multibody dynamics (see e.g. [17-19]). Flexible multibody dynamics is a rapidly growing branch of computational mechanics and many industrial applications can be solved using newly proposed flexible multibody dynamics approaches. Studied problems are characterized by a general large motion of interconnected rigid and flexible bodies with the possible presence of various nonlinear forces and torques. There are many approaches to the modelling of flexible bodies in the framework of multibody systems [20]. Comprehensive reviews of these approaches can be found in [17] or in [21]. Further development together with other multibody dynamics trends was introduced in [22]. Details of multibody formalisms and means of the creation of equations of motion can be found e.g. in [23, 24].

The simplest way how to incorporate fibres in equations of motion of a mechanism is the force representation of a fibre (e.g. [25]). It is assumed that the mass of fibres compared to other moving parts is low to such an extent that the inertia of fibres is negligible with respect to other parts. The fibre is represented by the force dependent on the fibre deformation and its stiffness and damping properties. This way of the fibre modelling is probably the most frequently used one in the cabledriven robot dynamics and control (e.g. [26, 27]). The fibre-mass system fulfils all requirements for modelling the fibre using the force representation of the fibre. A little more precise approach is based on the representation of the fibre by means of a pointmass model (e.g. [28]). It has the advantage of a lumped point-mass model. The point masses can be connected by forces or constraints.

The massless fibre model is considered in this phase of investigation of the WFPD system. The fibre model is considered to be phenomenological and it is modelled by the forces which comprise, e.g., influences of fibre transversal vibration, jumping from pulley, etc. The multibody models of the

WFPD system referring to the symmetric and asymmetric position of the weight with respect to the plane of drive-pulley symmetry slightly differ [4]. In the case of symmetric position, the number of degrees of freedom in kinematic joints is 5 (in the case of asymmetric position, the number of degrees of freedom in kinematic joints is 6). The weight (or the weight with added mass), the pulley and the drive are considered to be rigid bodies. A planar joint between the weight and the base (prismatic linkage), a revolute joint between the pulley and the base and a prismatic joint between the drive and the base are considered. Clearly, the movement of the drive is kinematically prescribed. Behaviour of this nonlinear system is investigated using the alaska simulation tool [9].

4 Simulation and experimental results

As it has already been stated, the main aim of simulations was to create a phenomenological model of a fibre. In order to look for compliance of the results of experimental measurement with the results of simulation influences of the fibre stiffness and the fibre damping, the coefficients are considered. The friction force acting between the weight and the prismatic linkage was considered to be a phenomenological model parameter in the first phase of investigation [13].



Figure 3. Friction force acting between the weight and the prismatic linkage.

Investigation of the (carbon) fibre properties by eliminating the influence of the drive and of the pulley was an intermediate stage before the measurement on the stand [13]. A phenomenological model dependent on the fibre stiffness and fibre damping coefficient was the result of this investigation. The focus was on the fibre model that ensures the greatest possible coincidence not only with time histories of the weight displacement but also with the dynamic force acting in a fibre. Fibre stiffness and a fibre damping coefficient were considered to be constant in this phase of the fibre behaviour research. The nonlinear friction force course depending on the weight velocity and determined (especially on the basis [29] and [30]) at investigating the weight-fibre mechanical system [13] with the angle of inclination of the inclined plane $\alpha = 30$ degrees is applied to the model of the WFPD mechanical system [4-8] (see Fig. 3). Values of fibre stiffness ($c = 34 \cdot 10^3$ N/m) and fibre damping coefficient (see Table 2) were calculated on the basis of the values

When simulating the experimental measurements for a "quicker" drive motion [4-6], the local extremes of the time histories of measured and calculated weight displacement were more or less different. These results show that the parameters of the fibre phenomenological model must be considered, in addition, dependent on the velocity of the weight motion. That is why the influence of the velocity-dependent damping "coefficient" considered in the fibre model and exerted on dynamic response of the system is investigated.

determined in [4].

Velocity-dependent damping "coefficient" b of the fibre is supposed to be in the form:

$$b = \begin{cases} b_{\rm c} , & \text{if } v \le v_{\rm tr} \\ b_{\rm c} + (v - v_{\rm tr}) \cdot b_2 , & \text{if } v > v_{\rm tr} , \end{cases}$$
(1)

where $b_{\rm c}$ is the constant fibre damping coefficient (taken from [4]), b_2 is the constant, v is the instantaneous velocity of the weight and v_{tr} is the threshold value of the velocity of the weight [7, 8]. The optimal (constant) value of constant b_2 is found. Threshold value of weight velocity $v_{\rm tr}$ was found in [7] and confirmed in [8]. The influence of the fibre damping "coefficient" values exerted not only on time histories of the weight displacement but also on the dynamic force acting in the fibre is evaluated partly visually and partly on the basis of the value of the correlation coefficient between the records of the experimental measurements and the simulation results. This approach based on the calculation of the statistical quantities and applied in order to express directly the relation between two time series has appeared to be suitable for

comparing two time series in various cases - e.g. [31].

Correlation coefficient $R(\mathbf{p})$ [32] defined for two discrete time series $y^{(1)}$ (the time history recorded at experimental measurement) and $y^{(2)}$ (\mathbf{p}) (the time history aimed at simulation with the multibody model and the function of investigated parameters \mathbf{p}) was calculated:

$$R(\mathbf{p}) = \frac{\sum_{i=1}^{n} \left(y_{i}^{(1)} - \mu_{1} \right) \cdot \left[y_{i}^{(2)}(\mathbf{p}) - \mu_{2}(\mathbf{p}) \right]}{\sqrt{\sum_{i=1}^{n} \left(y_{i}^{(1)} - \mu_{1} \right)^{2} \cdot \sum_{i=1}^{n} \left[y_{i}^{(2)}(\mathbf{p}) - \mu_{2}(\mathbf{p}) \right]^{2}}},$$
(2)

where, μ_1 and $\mu_2(\mathbf{p})$ are mean values of the appropriate time series. The maximum value of the correlation coefficient is 1. The more the compared time series are similar to each other, the more the correlation coefficient tends to 1. The advantage of the correlation coefficient is that it quantifies very well the similarity of two time series by scalar value obtained by using a simple calculation.

The problem can be put as the problem of the minimization of the objective function in the form:

$$\psi(\mathbf{p}) = (1 - R(\mathbf{p}))^2. \tag{3}$$

Table 2 presents the optimal values of parameters in Equation (1) of the investigated model of the WFPD system. In the second and the third columns of Table 3, time histories of the weight displacements

u are compared using correlation coefficient $R(\mathbf{p})$, whereas in the fourth and the fifth column of this table time histories of the dynamic forces acting in the fibre are compared. From Table 3 it is evident that the values of correlation coefficient $R(\mathbf{p})$ are better for the determined velocity-dependent damping "coefficient" than for the constant damping coefficient in the fibre model. In the time histories of the dynamic force acting in fibre, the correlation coefficient $R(\mathbf{p})$ improvement is not evident very much since the values are of rather informative character. As already stated, in fibre phenomenological model, the forces act in fibre longitudinal direction only. Their effect also comprises e.g., influences of fibre transverse vibration, jumping from pulley etc. That is why time fibre histories of fibres acting in the phenomenological model cannot comply with time histories of the measured forces acting in the fibre longitudinal direction. Correlation coefficient $R(\mathbf{p})$ mentioned in Equation (2) does not have any significant information capacity for comparing time histories of dynamic force acting in the fibre. At time histories of dynamic force acting in the fibre during determining the optimum values of velocitydependent damping "coefficient", the special attention was paid to keeping the character of their course and achieving the best possible agreement of extreme values of the measured and the calculated dynamic forces.

Table 2. Values of damping coefficients of the fibre model

	Threshold value of the	Damping coefficients	
	velocity v_{tr} [m/s]	$b_{\rm c}$ [N·s/m]	<i>b</i> ₂ [N]
Constant coefficient	-	27.5	-
Velocity-dependent damping "coefficient"	0.4	27.5	385

Table 3. Values of correlation coefficient R(p)[-]

	Comparison of the time histories of the		Comparison of the time histories of the dynamic	
	weight displacement		force acting in fibre	
Tested	Constant damping	Velocity-dependent	Constant damping	Velocity-dependent
situation	coefficient	damping "coefficient"	coefficient	damping "coefficient"
2	0.9929	0.9934	0.3904	0.3847
3 c	0.7552	0.9190	0.03925	0.2084
7 a	0.9999	0.9999	0.5673	0.5743
10	0.9834	0.9838	0.5078	0.5142
11	0.2187	0.6890	0.2323	0.1146



Figure 4. Time history of weight displacement in a "quicker"- tested -situation, a trapezoidal periodic exciting signal at frequency of 1.25 Hz, and the weight without added mass.







Figure 6. Time history of weight displacement in a "quicker"-tested-situation, a trapezoidal periodic exciting signal at frequency of 2 Hz, and the weight without added mass.



Figure 7. Time history of dynamic force acting in a fibre in a "quicker"-tested-situation, a trapezoidal periodic exciting signal at frequency at 2 Hz, and the weight without added mass.



Figure 8. Time history of weight displacement in a "quicker"-tested-situation, a trapezoidal periodic exciting signal at frequency of 1.25 Hz, and the weight with added mass.



Figure 9. Time history of dynamic force acting in a fibre in a "quicker"-tested-situation, the trapezoidal periodic exciting signal at frequency of 1.25 Hz, and the weight with added mass.



Figure 10. Time history of weight displacement in a "quicker"-tested-situation, the trapezoidal periodic exciting signal at frequency of 2 Hz, and the weight with added mass.



Figure 11. Phase plane (weight velocity versus weight displacement) in a "quicker"tested -situation, the trapezoidal periodic exciting signal at frequency of 2 Hz, and the weight with added mass (part of the record).



Figure 12. Time history of dynamic force acting in a fibre in a "quicker"-tested-situation, a trapezoidal periodic exciting signal at frequency of 2 Hz, and the weight with added mass.



Figure 13. Time history of weight displacement at a "slower"-tested-situation, the quasisinusoidal periodic exciting signal at frequency of 0.35 Hz, and the weight without added mass.



Figure 14. Phase plane (weight velocity versus weight displacement) in a "slower"tested-situation, a trapezoidal periodic exciting signal at frequency of 2 Hz, and the weight with added mass (part of the record).



Figure 15. Time history of dynamic force acting in a fibre at a "slower"-tested- situation, a quasi-sinusoidal periodic exciting signal at frequency of 0.35 Hz, and the weight without added mass (part of the record).

At "slower" drive motions, time histories of the weight displacement recorded in the experimental measurements and computed in computer simulations are approximately identical (see [4-6] and Fig. 13) and hence it is not desirable to change values of parameters of the fibre phenomenological model, i.e., it is not necessary to consider the parameters of the fibre phenomenological model to be velocity-dependent). It is evident that this was the very reason why the threshold value of weight velocity v_{tr} was determined in such a way that the maximum velocity of the weight during "slower" drive motions should be lower than v_{tr} (see Table 2).

General pieces of knowledge obtained during investigating the WFPD system, independently of the combination of the symmetric/asymmetric position of the weight [4] and of the weight without/with added mass [7], are similar.

As it has already been stated in [5-7], the highest frequency of drive motion, i.e., the highest frequency of input signal at investigation of the WFPD system is 2 Hz (see Fig. 6 and Fig. 10). This frequency of drive motion is much lower than natural frequencies of the computer model of a linearized system in an equilibrium position. Natural frequency corresponding to the weight vibration of the system with weight without added mass is 25 Hz and natural frequency of the system with weight with added mass is 15.25 Hz. It means that in case of weight vibration in "quicker" - tested - situations, this paper does not concern with excitation of resonant vibrations. As already stated, vibrations given by strongly nonlinear behaviour of fibres able to transfer only tensile force are involved since in compression they are not able to transfer any force, which can even have the character of chaos..

In the "quicker"-tested-situations, the measured and the computed time histories of the weight displacement are of the same character (see Fig. 4, Fig. 6, Fig. 8 and Fig. 10).

On the basis of results it is evident that the velocitydependent damping "coefficient" (see Fig. 4, Fig. 6, Fig. 8 and Fig. 10), the velocity-dependent stiffness of the fibre in the model (not published) and especially both velocity-dependent stiffness and velocity-dependent damping "coefficient" [7] have a great contribution to the improvement of agreement of the measured, computed time histories and the local extremes of the weight displacement. Phase plane diagram in Fig. 11 (weight velocity versus weight displacement) at a "quicker"-testedsituation shows that the velocity-dependent damping "coefficient" leads to a decrease in the weight displacement. In a "slower"-tested-situation (see phase plane diagram in Fig. 14), the velocitydependent damping "coefficient" is not employed. As already stated in [4-8] in all the simulations the computational when changing model parameters, the time histories of dynamic force acting in the fibre are more or less different but their character remains the same. From Fig. 5, Fig. 7, Fig. 9, Fig. 12 and Fig. 15 it is evident that time histories of dynamic force acting in the fibre are not suitable for determining the parameters of the fibre phenomenological model. It follows from the mentioned fact that the phenomenological model of the fibre is to cover, among others, influences of the fibre transverse vibrations, loss of contact with the pulley etc. Although those phenomena are not physically included (but by the change in the already introduced model parameters), it is evident that the time histories of the dynamic force acting in the fibre cannot be expected to be of the same course.

To search the parameters of the fibre phenomenological model, it is necessary to use the results of experimental measurements with the a little "quicker" drive motion. The possibility of performing experimental measurements with other time histories of drive motion or with a different geometrical arrangement of the experimental stand will be analysed.

5 Conclusions

The approach to the fibre modelling based on the force representations was utilized to investigate the weight motion in the weight-fibre-pulley-drive (WFPD) mechanical system. The aim of simulation is to create a phenomenological model of the fibre utilizable in fibre modelling in the case of more complicated mechanical or mechatronic systems. The created phenomenological model is assumed to be dependent on the velocity-dependent fibre damping "coefficient".

Development of the fibre phenomenological model will continue. More sophisticated phenomenological models of the fibre are supposed to consider more complicated dependencies of the fibre stiffness and fibre damping "coefficients" on the weight velocity.

In addition, it must be stated that the model of the fibre-pulley contact appears to be problematic in the computational model.

Note: The paper is a modified version of the contribution [8].

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